Renaissance Man Meets the Pin Factory:
A Theory of Diverse Specialization under Uncertainty

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Abstract

SARAH F. RILEY: Renaissance Man Meets the Pin Factory:
A Theory of Diverse Specialization under Uncertainty.
(Under the direction of Gary Biglaiser.)

The fraction of university students in the US graduating with multiple majors or joint degrees has been increasing for at least the last decade, but this trend has not yet been explained. I demonstrate theoretically that innate ability differences are sufficient to create divergence in observed skill investment patterns across individuals and that changes in uncertainty can contribute to changes in the fraction and types of students who acquire more than one skill before entering the skilled labor force for the first time. I assume that the level and balance of abilities constrain the investment possibilities open to students. I find that only in the presence of significant cost complementarities, in the forms of economies of scale or learning by doing, are the brightest and most balanced students the most likely to acquire two marketable skills. This case occurs when the cost per skill falls sufficiently that the marginal surpluses from skill level and skill diversity become positive for the most balanced individuals, who face the greatest absolute skill cost, while perceived uncertainty is sufficiently high to provide the most generally able individuals with an incentive for acquiring more than one skill. After this theoretical exploration, I consider the abilities and skill investment patterns of a group of recent bachelor's degree recipients from UNC-Chapel Hill. The results of this empirical analysis suggest that abilities at matriculation play a far more important role than financial or other demographic factors in determining skill investments, and that cost complementarities are present in the skill acquisition process. The most generally able students in this sample are the most likely to double major, but neither the most able nor the most balanced make the most diverse undergraduate investment. In combination, the results of these analyses suggest that the fraction of students with a double major has been increasing over time within

institutions of higher learning because (1) the mean level of general ability of admitted students has been increasing from year to year, and (2) increased perceived wage uncertainty has encouraged the most able students to diversify their skills by adopting a second major.

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Chapter 1

Introduction

1.1 Motivation and Research Questions

Human capital diversification is generally very costly. Nevertheless, some individuals do become highly skilled in a variety of fields, even before entering the work force. In fact, such skill diversification is becoming increasingly popular in the US among highly educated workers, who frequently graduate from college with multiple majors or participate in joint professional degree programs. For example, Georgetown University and Washington University in St. Louis saw the fractions of their students graduating with multiple majors or degrees rise from 14% in 1996 to 23% in 2002, and from 28% in 1997 to 42% in 2001, respectively.[1] Similarly, the fraction of students graduating with multiple undergraduate majors from UNC-Chapel Hill increased from 20% in 1999 to 28% in 2006.[2] Thus, large numbers of the highly educated are finding skill diversification quite feasible.

Some of these students appear to be obtaining multiple credentials in response to what they perceive as recent increases in the competitiveness and uncertainty of the skilled labor market. For example, one college student quoted in a 2002 article in the *New York Times*[3] concerning

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[1]See the November 17, 2002, *New York Times* article entitled *For Students Seeking Edge, One Major Just Isn't Enough.*

[2]These percentages come from my analysis data sample, which has been refined somewhat from the UNC database of all graduates during this time period.

[3]Ibid.

the recent growth in the fraction of college students choosing double and triple majors in the US went so far as to state, "I'm hedging my bets. The more fields I am prepared or qualified to work in the less I have to worry about problems in any given industry." Furthermore, an academic administrator interviewed for the same article acknowledged a growing trend: "I think students are increasingly aware that they might have more than one career, that they might need expertise in a variety of areas."[4] In short, a self-insurance motive seems to drive the choice of multiple specialties in some cases.

At least two factors have likely contributed to such student perceptions and behavior. First, the relative likelihoods of employment of educated and uneducated workers in the US have been converging since the middle of the last century,[5] suggesting that a higher level of education is no longer the relative job "insurance" that it once was. This trend may have contributed to a perception by students that it is no longer enough simply to attain a high level of education: breadth is also requisite for sustained skilled employment. Second, wage uncertainty among college graduates has also been increasing over this period. Common explanations for this increase in skilled wage dispersion include technological change and outsourcing, a relative increase in the supply and heterogeneity of college graduates, changes in industry and firm structure, and a shift toward more flexible or insecure employment conditions, including part-time work.[6] The popular media, in particular, has done much to instill the idea that outsourcing may play a large role in undermining US job and wage security for skilled workers. To the extent that highly educated workers view this change in wage dispersion as an increase in the idiosyncratic risk associated with their expected wages following graduation, it is reasonable that these workers should attempt to diversify their employment opportunities

[4] The article also suggests that some students are majoring in several fields because it is easier for them to do so than in previous years. That is, more students are entering college with Advanced Placement or International Baccalaureate credit, which frequently may be applied to satisfy general distribution requirements, and thus have more flexibility in selecting their coursework.

[5] See Aaronson and Sullivan (1998), Gardner (1995), Boisjoly, Duncan, and Smeeding (1998), Polsky (1999), Stewart (2000), Farber (2001), Rodriguez and Zavodny (2003), and Helwig (2004).

[6] See Autor, Katz, and Kearney (2005); Gottschalk and Hansen (2005); McCall (2000); Hoxby and Terry (1999); Pryor and Schaffer (1997); DiNardo, Fortin, and Lemieux (1996); Katz and Murphy (1992); and Levy and Murnane (1992).

by acquiring expertise in multiple fields.

Nevertheless, the majority of students in higher education continues to specialize in only one area. Therefore, it is natural to ask what is different about those students who do choose to diversify. In this paper, I demonstrate theoretically that innate ability differences[7] are sufficient to create divergence in observed skill investment patterns across individuals and that changes in uncertainty in this context can contribute to changes in the fraction and types of students who select more than one skill.

In particular, individuals differ both in the level of their innate general cognitive ability and in the balance of their innate specific cognitive abilities. For example, some individuals learn faster and are able to grasp abstract concepts more readily than others; such individuals are more generally able. Similarly, some individuals have natural talent for both mathematics and languages in equal measure, while others have a definite predisposition for one area of study or the other; such individuals are more balanced in their specific abilities. These two types of innate ability difference are in part responsible for variation in the number of skills that highly educated individuals acquire prior to entering the skilled labor market for the first time.

More formally, evidence from psychology suggests that two major categories of cognitive ability, or "intelligence," are likely to have impacts on educational achievement. The first, and more salient, of the two is the general intelligence factor g. This latent variable has the colloquial interpretation of an individual's overall ability to learn and to engage in higher-order thinking skills. For example, we may associate abstract reasoning ability, as well as rapidity of thought, largely with the factor g. This factor typically explains a majority of the predictable variation in scholastic achievement test scores.[8] The second category of cognitive ability that should impact educational achievement comprises the group factors that are associated with

[7] "Innate cognitive abilities" need not necessarily refer to cognitive abilities at birth, exclusive of environmental and developmental factors. Rather, these may simply be the abilities that are realized, or "crystallized," prior to college entry.

[8] See Jensen (1998) and Ree and Carretta (1994).

g. Examples of group factors commonly identified in this context include verbal, mathematical, and analytic reasoning abilities. [9] Such specific cognitive factors typically explain the remaining predictable variation in tests of academic achievement.

Thus, an individual's *level* of general cognitive ability partly reflects his overall potential for learning skills at greater *depth*. Similarly, an individual's *balance* in specific cognitive abilities should partly reflect his relative potential for learning a greater *variety* of specific skills. Therefore, a lack of ability in either of these two dimensions may place limits on the extent to which an individual finds either specialization or diversification worthwhile.

However, the way in which individual abilities interact with perceived uncertainty to determine skill number choice has not yet been considered, and it is not obvious a priori that more generally able or more balanced individuals should necessarily be the most likely to specialize in more than one skill area in this situation. Because individuals who are more generally able are also more highly compensated for a given level of education and are also more likely to be employed, it may be that the expected benefit from investing in one skill is sufficiently high to offset the diversification incentive provided by uncertainty; in this case, those of higher general ability would be less likely than their less able peers to invest in a larger number of fields. Similarly, individuals who are more balanced in their specific abilities may have more career options for any given level of skill, and this wider range of available choices may deter educational investment through the mechanism of anticipated regret[10]. These individuals may also find any given skill more costly because of a weaker innate skill preference. In this case, more balanced individuals would be less likely than their less balanced peers to invest in any one skill, not to mention two or more.

With these observations in mind, I characterize more specifically how abilities and perceived uncertainty interact to determine whether an individual should optimally choose to diversify before entering the skilled labor market for the first time. I employ a spatial, analytic model that incorporates ability constraints in two dimensions and allows skill choice in

[9] Alternatively, academic and technical group factors may arise, as has been the case in analyses of the ASVAB. See Ree and Carretta (1994)

[10] See Schwartz (2004).

three dimensions: an individual of a particular balance of specific abilities and a particular level of general ability chooses the number, level, and diversity of his skills for a given level of exogenous, diversifiable wage uncertainty. The model is designed primarily to capture how abilities operate through costs to determine skill acquisition patterns. Specifically, I assume that general ability constrains individuals because it is costly to over- or under-invest in education level. In addition, I assume that specific-ability balance constrains individuals through the psychic cost of anticipated regret and weaker affinity for any specific skill. This model allows me to characterize which individuals are most likely to select more than one skill at various levels of uncertainty. After this theoretical exploration, I then consider empirically the relationship between the abilities and skill investment patterns of a group of recent bachelor's degree recipients from the University of North Carolina at Chapel Hill.

As should be clear from the above discussion, I take an interdisciplinary (and slightly behavioral) approach to analyzing the relationship between ability and skill investment. This choice is appropriate for two reasons. First, the psychic costs, as well as the economic costs, arising from ability differences are likely to impact skill investment decisions. Therefore, I assume that the cost associated with general ability may include anticipated non-wage job dissatisfaction resulting from over- or under-investment in education level, in addition to the obvious economic costs of foregone wages or excess tuition expenditures that would also be relevant in these cases. Similarly, I assume that the cost deriving from greater specific-ability balance includes the anticipated regret resulting from having too many skill investment options. Second, and more important, the concept of ability means many different things to different people: some interpret it as a form of cognitive competence, while others see it as all-inclusive of any behaviors and/or personality traits that contribute to success. To avoid confusion, therefore, I define ability very narrowly, using latent variable concepts from psychometrics, such as g and its associated group factors. Analogously precise definitions of ability do not exist in economics.

1.2 Contributions to the Literature

To the best of my knowledge, this analysis departs from existing theoretical economic literature in at least three respects. First, I consider the joint impact of both ability and perceived uncertainty on skill investment. Existing work considers the isolated impact of either ability[11] or uncertainty[12] on human capital acquisition, but not both.[13] However, uncertainty may encourage some individuals to invest in multiple skills while discouraging others.

Second, I consider ability as two distinct endowments. These endowments operate in independent dimensions to influence the costs of skill acquisition. In contrast, existing work generally considers ability as a one-dimensional endowment that is perfectly transferable across intensive and extensive investment margins.[14] However, such an assumption is restrictive. Many different types of ability exist that do not necessarily translate into universal competence.

Third, I consider skill choice in three dimensions. Existing work considers the choice of skills in at most two dimensions, namely level and breadth, or level and number. In reality, however, individuals must choose the number, level, and breadth of their skills. Moreover, the distinction between skill number and skill breadth, or diversity, is important. Acquiring two skills is more beneficial from a standpoint of labor demand diversification when those skills have little in common.

In addition, this paper contributes to the empirical economic literature on educational investment by considering which student characteristics influence the decision to double major. A wealth of papers has considered the role of ability in determining educational attainment

[11] Weiss (1971) discusses the impact of a one-dimensional ability measure on the time spent in school. Baumgardner (1988) considers the impact of the productive endowment of physicians on their depth and breadth of service. Lazear (2003, 2004, 2005) examines the effects of specific-ability balance and entrepreneurial talent on the level and number of skills acquired by entrepreneurs.

[12] Levhari and Weiss (1974) and Eaton and Rosen (1980) consider the impact of uncertainty on skill level investment. Grossman and Shapiro (1982) and Murphy (1986) examine the impact of uncertainty on the choice of general versus specialized human capital.

[13] Kim (1989), Murphy (1986), and Baumgardner (1988) also examine the impact of the extent of the market on the level and breadth of investment/provision. Johnson (1979) considers the impact of ex post occupational mobility on the demand for breadth of training.

[14] A notable exception is the work of Lazear (2003). (See also Lazear (2004) and Lazear (2005)). Lee (2005) extends Lazear's work to occupational categories other than entrepreneur.

and the returns to education, as well as selection into a specific single major field. However, I am aware of only one paper that investigates the implications of choosing a second major[15], and that investigation focuses on the wage returns to such an investment after graduation, rather than on the undergraduate skill investment process.

1.3 Key Results and Policy Implications

The basic theoretical framework suggests that all individuals specialize fully under low perceived uncertainty. The relative benefit from diversification is small, and individuals acquire a higher level of skill when uncertainty is low. Therefore, the benefit to skill investment is highest under low uncertainty, and all individuals find it worthwhile to acquire a skill, but there is insufficient incentive to invest in more than one.

As perceived uncertainty increases, individuals of lowest general ability and greatest ability balance are the first to respond to increases in uncertainty by choosing to acquire no skills, or to opt out of the skilled labor market. This result stems from the facts that these individuals (1) acquire a low level of skill, which translates into a relatively low return, and (2) find skill acquisition very costly: this combination provides a strong disincentive for skill investment.

Extremely high levels of uncertainty discourage skill acquisition by any but the least balanced. More generally able individuals over-invest in skill level and face a higher cost from that investment; as a result, they are less likely to diversify, given that they choose to acquire one skill.

The implications of the model become less stylized and more intuitive when the cost per skill is allowed to vary with the number of skills acquired. In this extended model, the set of individuals who select more than one skill varies with both the level of uncertainty and the degree of cost complementarity and may include or favor the most balanced and most generally able individuals.

Under high perceived uncertainty, individuals of lesser specific-ability balance are most likely to acquire multiple skills. However, those of greater specific-ability balance may be more

[15] See Del Rossi and Hersch (2008).

likely than those of intermediate specific-ability balance to acquire multiple skills, depending on the extent to which the cost per skill falls as the number of skills acquired increases. Also in this case, individuals of higher general ability are more likely to acquire a positive number of skills but may be either more or less likely than less able individuals to specialize fully, depending on the extent to which the cost per skill falls as the number of skills acquired increases.

More generally, the intuition underlying skill investment patterns for all individuals is as follows. The least balanced are the most likely to face a positive marginal surplus from skill diversity. In the presence of a significant cost complementarity, the set of those with a positive marginal surplus from diversity expands to include the most balanced individuals. Those of intermediate specific-ability balance follow as the cost complementarity increases. Furthermore, all individuals generally face a negative marginal surplus from level in the absence of a significant cost complementarity. However, as the cost complementarity increases, this marginal surplus becomes positive for all individuals. The absolute value of the marginal level surplus term increases with general ability. Therefore, more generally able individuals are more likely to prefer a second skill under a significant cost complementarity and less likely to prefer a second skill in the absence of one, relative to their peers in specific-ability balance, and for a given level of perceived uncertainty. Finally, individuals of greater specific-ability balance will tend to prefer a greater number of skills than before as the cost per skill falls.

In conjunction with these theoretical inferences, the empirical results suggest that cost complementarities are present for UNC graduates. More generally able students are more likely to graduate with two majors, though ability balance does not appear to play an important role in determining the number of majors for this sample. However, the analysis also indicates that the most generally able students and the most balanced students are likely to be among the most specialized in their undergraduate investments, as measured by the average number of courses that they take in any given department. As a result, greater ability in either dimension does not necessarily translate into a more diverse acquired skill set.

Thus, the empirical results suggest that ability balance may be much less important than general ability in determining the number of high-level, marketable skills that students acquire, because the most generally able individuals benefit most from cost complementarities in the skill acquisition process. However, these results also suggest that the theoretical intuitions that more generally able individuals have less incentive to invest broadly and that the most balanced individuals may make a more specialized investment appear reasonable in the more general context of all the courses that students choose to take while in school. Therefore, the theoretical model appears to capture at least some of the trade-offs that UNC students consider in practice.

In addition, the empirical results suggest that changes in economic conditions have played an important role in determining the fraction of students graduating with a second major. Cohort effects, which should proxy for perceived uncertainty and other labor market factors, have increasingly contributed to the likelihood of a double major over time.

Because both abilities and perceived uncertainty contribute to the likelihood of a double major, a reasonable explanation of why the fraction of students in universities across the country graduating with multiple major or degrees has been increasing recently involves two factors. The results of the theoretical and empirical analyses presented here suggest that this trend can be explained as resulting from both an increase in mean SAT scores over time and the response of the most generally able students to increases in their perceived future wage uncertainty. The most generally able benefit most from cost complementarities and are thus most likely to invest in multiple skills when given sufficient incentive in the form of anticipated labor market uncertainty.

Finally, the empirical analysis suggests that men and women may have fundamentally different skill investment patterns. Both genders are more likely to acquire two majors when they are more generally able, but the brightest women will tend to acquire two majors that are increasingly similar, while the reverse is true for the smartest men. Women tend to make a more diverse investment overall, and only for them is ability balance an important determinant of their overall degree of specialization. These interesting differences may very well reflect or predetermine observed gender differences in labor market outcomes.

This analysis should be of interest to educated workers, educators, and politicians. It suggests that recent changes in the skilled labor market have caused some students to change the way that they invest in skills at the undergraduate level, and that abilities have played a considerable role in determining which students are best able to adapt to such changes. In particular, this analysis suggests not only that students of higher abilities are more likely to weather labor market risk, by virtue of their greater tendency to invest in multiple skills at a high level, but also consequently that a double major or joint degree is likely to be a strong market signal of ability in the same way that a graduate degree is a signal of ability. This selection among students may influence which job candidates employers prefer. Moreover, because students tend to select into more- or less-lucrative majors or major combinations based in large part on their abilities at matriculation, and because these abilities appear to constrain the skill investment options that are available to students, policy should encourage students to improve their fundamental math and verbal skills before they enter college.

1.4 Layout of the Paper

The next two chapters present the theoretical model and an overview of the theoretical results. Chapters 4 and 5 similarly present the empirical framework, discuss the data, and then present the empirical results, along with a discussion of the implications of the theory and the empirical results for UNC students. Chapter 6 concludes and suggests directions for both policy and future research.

Chapter 2

Theoretical Framework

2.1 Basic Model

2.1.1 Overview

The model is the optimization problem of an individual whose objective function has three independent components:

- a benefit from investment in skill level that increases with general ability and decreases with uncertainty
- if more than one skill is acquired, a bounded benefit from investment in skill diversity that depends inversely only on the similarity of these skills
- a quadratic cost from skill investment that increases in the number, level, and diversity of skills acquired, and that depends on the ability profile of the individual

In particular, to allow for closed-form solutions, I assume that both the total benefit from skill investment and the total cost from the same are separable in the individual's choice variables. In consequence, it is also possible to think of the overall objective function, or total surplus, of the individual as simply the sum of two independent expressions:

- the surplus from skill level (benefit less cost in the "vertical" skill dimension)
- the surplus from skill diversity (benefit less cost in the "horizontal" skill dimension)

The individual maximizes his total surplus from skill acquisition by choosing the characteristics of his skills. In particular, he chooses their number, level, and diversity, given the level of his perceived uncertainty. He makes these choices and acquires these skills before he enters the skilled labor market.

2.1.2 Skills

Individuals may acquire zero[1], one, or two skills. The space of acquirable skills is represented by the outer surface, above the (x, y)-plane, of a cylinder of radius 1, which is illustrated in Figure 2.1.5.

In particular, for any given skill level (λ), which increases continuously[2] along the z-axis, skills form a continuum around the circumference of the unit-circle cross-section of the cylinder at that level. For example, at the level of a bachelor's degree from a particular institution, the various majors may be placed on the perimeter of the circle in such a way that the proximity of various skills to one another indicates their diversity, or degree of similarity. In other words, biology and chemistry may fall relatively close together on the circle, while biology and the Chinese language may be rather farther apart. Skills diametrically opposed are the least similar.

2.1.3 Individuals

Individuals are located within the cylinder and are characterized by three parameters:

- specific-ability balance (v)
- skill preference (θ)

[1] Individuals acquiring zero skills presumably take unskilled employment if they participate in the labor force.

[2] The assumption of a continuous choice space derives from the observation that even educational institutions providing the same nominal credentials, such as the bachelor's degree, vary greatly in the quality of the degrees they provide. For example, an A.B. from Harvard is not the same as a B.A. from Kalamazoo College. Therefore, one may argue that individuals effectively face a continuum of level choices when they select both their desired credential and which school to attend.

- general ability (g)

All other factors that may influence skill choice are assumed constant across workers and otherwise conducive to skill acquisition.

The skill preference parameter θ simply indicates an individual's most preferred skill; for example, an individual may have a strong natural interest in and facility for history. A high value of g corresponds to high general ability, but a high value of v indicates low specific-ability balance. Possible empirical interpretations of g and v are, respectively, general intelligence and balance in the associated intellectual group factors (e.g., verbal, mathematical, spatial, etc.); other real-world interpretations may be possible. Both v and g are normalized within the open interval (0,1), while θ varies between zero and 2π. In short, the location of the individual in three-space is given in Cartesian coordinates by $(v\cos(\theta),\, v\sin(\theta),\, g)$.

2.1.4 Cost of skill acquisition

I assume that an individual's cost of acquiring a particular skill is the square of the distance from that individual's location within the cylinder to that of his chosen skill on the cylinder's surface. More formally, making use of the distance formula for three dimensions, the cost C_{ij} of skill θ_{ij} of level λ_{ij} is given by

$$C_{ij} = (\cos(\theta_{ij}) - v\cos(\theta))^2 + (\sin(\theta_{ij}) - v\sin(\theta))^2 + (\lambda_{ij} - g)^2 \tag{2.1}$$

where the index $i = 1, 2$ denotes the total number of skills acquired by the individual and the index $j = 1, 2$ indicates one of these skills in particular. Moreover, two aspects of the assumed relationship between individual location and skill cost are worthy of note.

First, an individual of a given level of g finds it costly to under-invest in education level. For example, someone ordinarily destined to become a research biologist may find the work of a janitor unsatisfactory from the point of view of mental stimulation. Similarly, over-investment is also costly. For example, one may reasonably imagine that a student who struggles to complete a bachelor's degree may end up working in a position not requiring the credential.

Second, skill cost is increasing in specific-ability balance. That is, individuals near the center of the cylinder find it more costly to acquire their most preferred skill θ than do individuals who are less balanced and are located closer to the perimeter. However, more balanced individuals find all skills of a given level more similar in cost, and thus have a weaker innate skill preference, than do individuals who are less balanced. This trade-off between absolute and relative skill costs across individuals of differing specific-ability balance makes sense if one notes that more balanced individuals may face a higher cost of skill acquisition than their less balanced peers in general ability.

In other words, note that more balanced individuals have the potential to be competent at everything to a more similar degree than do their less balanced peers. Therefore, they are more likely to make mistakes when selecting the particular skills in which they invest, simply because they have more options. Moreover, anticipation of this fact may cause them to be indecisive or seek to avoid a decision entirely. For example, not only do more balanced individuals anticipate that the skill with the highest future wage realization is unlikely to coincide with any given skill that they will have chosen, but they also know that they have sufficiently broad abilities to find the acquisition of the future optimal skill quite feasible in the present, if they could only somehow know in advance what that will be. Thus, the specific-ability balance of an individual may be construed as the likelihood that he will regret any given skill investment decision. Such anticipated regret is a potentially large psychic cost.[3]

In short, the cost structure of the model is designed primarily to capture two ideas. First, an individual should find it least costly to acquire those skills for which he is best suited. In this respect, individuals find it costly to over-invest or under-invest in education level relative to their level of general ability, as well as to deviate greatly from their most salient skill preference, or talent. Second, the very existence of a choice among multiple alternatives may impose costs on the chooser. Specific-ability balance reduces the cost of acquiring less similar

[3] I ask the skeptical reader whether, when faced with a choice among a variety of mutually exclusive options, he or someone he knows has ever substantially delayed or even entirely avoided making a decision, simply for fear of making a mistake. Schwartz (2004) notes not only that anticipated regret frequently causes indecision or decision avoidance, but also that the anticipated regret of individuals is likely to be greater when they face a greater number of options.

skills but also raises the reference point relative to which any given skill investment must be evaluated.

2.1.5 Benefit of skill acquisition

I normalize the benefit from acquiring zero skills to zero, and I assume that the benefit B_i to the individual who acquires a positive number i of skills is

$$B_1 = \frac{g\lambda_{11}}{\sigma} \tag{2.2}$$

in the case that he acquires one skill[4], and

$$B_2 = \frac{g(\lambda_{21} + \lambda_{22})}{2\sigma} + \frac{\delta}{\pi} \tag{2.3}$$

in the case that he acquires two, where σ denotes perceived idiosyncratic labor demand uncertainty,[5] δ denotes skill diversity and corresponds to the length of the arc between the two skills on the circle, and $\pi \approx 3.14$ represents itself. The expression B_1 is simply the benefit from skill level investment for one skill. Similarly, the first term in the expression B_2 is the benefit to skill level investment when two skills are acquired, while the second term is the benefit from investment in skill diversity, or from the dissimilarity of the two skills. Moreover, three aspects of the assumed benefit functions are worthy of note.

First, I assume that the return to skill level investment is greater when the individual is more generally able. This assumption is justifiable in light of evidence suggesting that more generally intelligent individuals are more productive, more trainable because they learn faster, and have better job performance.[6] Thus, g may be viewed as a normalized productivity

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[4] Jensen (1998) suggests that an individual's developed general ability is theoretically the product of innate general ability (g) and education. Therefore, if we assume that the labor market compensates a worker in a manner commensurate with his level of developed ability, then it seems reasonable also to assume this multiplicative form of the market return to an individual's level of schooling.

[5] Unemployment corresponds to a wage of zero.

[6] See Jensen (1998).

parameter that is proportionately compensated.

Second, I assume that perceived uncertainty is constant across skills, so σ may be thought of as the average idiosyncratic wage dispersion across occupations. For example, if outsourcing and technological change were to cause the variance of compensation in every occupation to increase due to better sorting in the labor market, this would correspond to a global increase in idiosyncratic skill investment risk. Moreover, such an increase need not necessarily translate into increased systemic, or market, risk.[7] Thus, any given skill investment would be inherently more risky, but this risk would be partly diversifiable. In short, changes in σ are meant to capture exogenous, systemic changes in diversifiable uncertainty.

Third, the benefit from an individual's chosen skill diversity, the second term in B_2, is normalized relative to the maximum possible diversity, which occurs when $\delta = \pi$. The benefit from diversity thus falls between 0 and 1. As a result, skill diversity becomes relatively more important in the individual's benefit function as uncertainty increases but becomes relatively less so when employment outcomes are more certain. Moreover, implicit in the assumption that the benefit from diversity is bounded is the supposition that partial diversification of the risk associated with two particular skills can compensate only to a limited degree for the negative impact that such risk generally has on an individual's benefit from skill level investment. Finally, the particular normalization scale chosen for diversity, in conjunction with that for specific-ability balance, creates a situation in which the benefit from diversity will, for at least some individuals, cover the balance-derived cost of at most one skill. In other words, it seems reasonable that the surplus from skill diversity investment, in and of itself, should not provide all individuals with a sufficient incentive to acquire multiple skills in those cases in which the surplus from skill level investment is negligible.

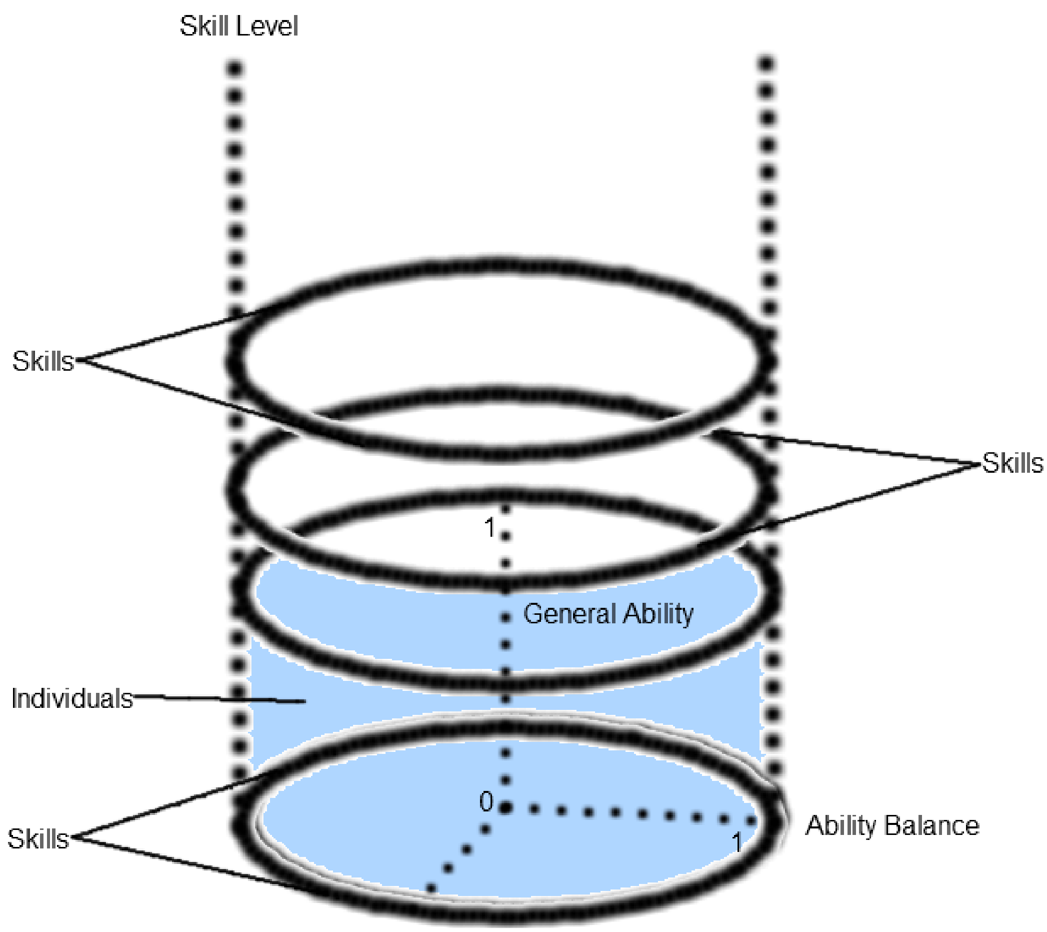
In the interest of model simplicity and tractability, I do not directly consider the mechanism by which workers find and retain employment. Rather, I intend the benefit portion of an individual's objective function to be a static, reduced-form representation of what is in reality

[7] On a related note, Campbell, Lettau, Malkiel, and Xu (2001) show for the case of the US stock market that the (weighted-) average level of idiosyncratic risk can change over time independently of any significant changes in market risk. Such an occurrence corresponds to a relative shift toward idiosyncratic risk in the determination of overall investment risk.

a dynamic and more complicated process. However, this formulation is not entirely unrealistic, in the sense that workers are unlikely to have specific and detailed knowledge about all of the various factors that may influence their future job and wage stability. Therefore, it is sufficient to note that uncertainty in the form of increased aggregate wage dispersion should decrease the expected benefit from employment when the worker is risk-averse, and that diversification in this context should be beneficial. Both of these effects are captured by the model. Finally, and more importantly, this simple benefit function allows focus to be placed explicitly on skill choice patterns as they depend on ability and perceived uncertainty.[8]

[8] In response to the questions of several of my colleagues, a few general comments about the origin of the model are in order. It is largely my own creation and derives only indirectly from any previous economic theory of which I am aware. That is, I was unable to find any established framework that could be adjusted easily to accommodate skill choice in three dimensions while also capturing the cost trade-off imposed by specific-ability balance. The choice of the functional form of the cost function and the choice of a spatial framework (as opposed to a non-spatial one) to represent the continuum of acquirable skills were admittedly inspired in part by the spatial product differentiation literature, including the work of Hotelling and Salop. However, the particular geometry chosen was selected primarily because I think that a circle, as compared with other geometric figures, best captures this trade-off. In addition, the benefit function reached its current state after I made many attempts to allow for interdependence between the benefits from level and diversity of skill investment; it was my experience that such interdependence generally precludes closed-form solutions.

Figure 2.1: Individuals and Skills



2.2 Extended Model

2.2.1 Overview

In the basic model outlined above, the cost per skill is independent of the number of skills acquired. In particular, the individual optimally choosing two skills in the context of the basic model chooses the level and diversity of his skills to maximize his surplus, which is given by

$$S_2 = B_2 - C_{21} - C_{22} \tag{2.4}$$

where S_i denotes the surplus from acquiring i skills.

However, the observation has frequently been made that skill acquisition exhibits complementarities. In other words, those individuals who invest in non-compulsory education tend to do so repeatedly. Moreover, the more educational investment a given individual has already made, the greater the likelihood that he will make further such investments. In short, something about the learning process feeds on itself in such a way that learning leads to learning; human capital is its own input and enhances the productivity of its own production.[9]

With this observation in mind, the extended model allows for a cost complementarity across skills when more than one is acquired.[10] In particular, I assume that the cost function may change in each of two ways. First, the fixed cost of each skill may vary with the total number of skills acquired. Specifically, the cost per skill may be lower when two skills are acquired simultaneously, as in the cases of a double major or a joint degree program; this type of cost complementarity may be viewed as an economy-of-scope. Second, the cost of a given skill may vary with the order in which skills are acquired, if these skills are obtained in tandem. In

[9]For discussions and empirical evidence, see Jenkins, Vignoles, Wolfe, and Galindo-Rueda (2003) and Cunha, Heckman, Lochner, and Masterov (2005).

[10]The assumption of a unit radius in the basic model can be relaxed by considering an arbitrary radius r. Doing so reduces the relative difference in skill costs for individuals of different ability balance if $r < 1$. In this case, some of the most balanced individuals (those with $v \leq \frac{1}{2\pi r}$) will acquire more than one skill. However, for $r \geq 1$, none of the most balanced individuals prefer two skills to one. Relaxing the assumption of a unit radius in this way provides results that are very similar to those of the extended model, as r pre-multiplies components of the cost function, as does γ. Thus, the extended model can also be interpreted as essentially a relaxation of the unit radius assumption, for $r \leq 1$.

particular, if an individual becomes more adept at learning in the course of his initial skill acquisition, then a second skill may be less costly; this type of cost complementarity may be interpreted as an instance of learning-by-doing.

More specifically, the extended model incorporates a cost complementarity parameter $\gamma \in (0,1)$, which becomes relevant only when two skills are acquired. This parameter is inversely related to the extent to which the cost per skill falls as the number of skills acquired increases from one to two. Note in what follows that $\gamma = 1$ would correspond to the case of the basic model. Therefore, the extension provides a more general analysis than does the basic framework.

2.2.2 Economy-of-scope

If an individual acquires two skills, he obtains them simultaneously. In so doing, he chooses the level and diversity of these skills to maximize his surplus, which is now given by

$$S_2^A = B_2 - \gamma C_{21} - \gamma C_{22} \tag{2.5}$$

In this case, the cost of *each* skill is reduced by the complementarity.

2.2.3 Learning-by-doing

If an individual acquires two skills, he obtains them in sequence, one immediately after the other, but still prior to entering the skilled labor market for the first time. In so doing, he chooses the level and diversity of his skills to maximize his surplus, which is now given by

$$S_2^B = B_2 - C_{21} - \gamma C_{22} \tag{2.6}$$

In this case, only the cost of the second skill is reduced by the complementarity.

2.2.4 Interpretation of γ as linked to g

Many real-world interpretations of γ are undoubtedly possible. For example, overlap in the courses required for various degree programs might be one such interpretation. However, it seems reasonable, especially in the context of sequential choice, to ask whether γ might actually be inversely related to g, and thus variable across individuals. In particular, if we continue to interpret g as representing general intelligence, narrowly defined, then g is essentially a measure of an individual's ability to learn, to think abstractly, and to draw meaningful conclusions quickly. As such, it should be expected to impact the capacity of an individual to earn multiple credentials at once, and to learn how to learn.

I examine the implications of this possibility by reinterpreting the cases of economy-of-scope and learning-by-doing after letting $\gamma = 1 - g$. This modification of the model is interesting primarily because it relaxes the assumption of the independence of the two skill investment margins, namely level and diversity, with regard to the relevance of the ability endowments to each. A high level of general ability now influences the cost of diversity, as well as those of number and level.

Chapter 3

Analytic Results

3.1 Primary analytic results – skill number

In the course of deriving solutions to both the basic and extended models[1]

for representative individuals ($\theta = 0$), I characterize the optimal choice of skill number relative to each individual's location on the v-continuum. (Recall that $v \in (0, 1)$ is inversely related to specific-ability balance.) More specifically, I define several v-cutoffs, which are functions[2] of g and σ, that indicate the regions of v in which individuals prefer a specific number of skills.

3.1.1 Basic model

In the case of the basic model, the analysis indicates that it is appropriate to define the following five v-cutoffs:

- $\hat{v}_{01}$, where $0 \leq \hat{v}_{01} \leq 1$:

  Individuals prefer one skill to none if $v > \hat{v}_{01}$. Otherwise, they prefer zero skills to one.

- $\hat{v}_{12}$ and $\hat{v}_{21}$, where $\frac{1}{2\pi} \leq \hat{v}_{12} \leq \hat{v}_{21} \leq 1$:

  Individuals prefer two skills to one if $\hat{v}_{12} < v < \hat{v}_{21}$ Otherwise, they prefer one skill to

[1] Formal results and proofs are provided in Appendix I.

[2] Though it is straightforward to determine the behavior of these cutoffs, it is possible to provide a closed-form expression for only the first of those defined below. For this reason, I omit specific functional forms.

two.

- $\hat{v}_{20}$ and $\hat{v}_{02}$, where $\frac{1}{2\pi} \leq \hat{v}_{20} \leq \hat{v}_{02} \leq 1$:

  Individuals with $v \geq \frac{1}{2\pi}$ prefer two skills to none[3] if either $v < \hat{v}_{20}$ or $v > \hat{v}_{02}$. Otherwise, they prefer zero skills to two.

The first of these cutoffs indicates that less balanced (higher-v) individuals are more likely to prefer one skill to none. The second and third cutoffs (second bullet point) above indicate that individuals of intermediate to low specific-ability balance are most likely to prefer two skills to one, while the third and fourth cutoffs (third bullet point) above suggest that those of lowest and highest specific-ability balance are most likely to prefer two skills to none, given that they may at any time prefer two to one. Possible relative placements of these cutoffs along the v-continuum are illustrated in Figure 3.1.1.

It is noteworthy that the most balanced individuals, namely those with $v < \frac{1}{2\pi}$, *never* acquire two skills, as they never prefer two skills to one. Intuitively, this result obtains because the marginal surplus from level in the acquisition of a second skill is always negative in the basic model. For this reason, two skills are preferred to one only when the marginal surplus from diversity is sufficiently large to make the total marginal surplus positive, which never occurs for the most balanced individuals.

Furthermore, the behavior of these five cutoffs depends on g and σ as follows:

- $\hat{v}_{01}$ decreases in g and increases in σ:

$$\frac{\partial \hat{v}_{01}}{\partial g} < 0, \ \frac{\partial \hat{v}_{01}}{\partial \sigma} > 0$$

- $\hat{v}_{12}$ and $\hat{v}_{21}$ converge as g increases and σ decreases but diverge when the reverse is true:

$$\frac{\partial \hat{v}_{12}}{\partial g} > 0, \ \frac{\partial \hat{v}_{21}}{\partial g} < 0, \ \frac{\partial \hat{v}_{12}}{\partial \sigma} < 0, \ \frac{\partial \hat{v}_{21}}{\partial \sigma} > 0$$

[3] Individuals with $v < \frac{1}{2\pi}$ always prefer one skill to two. Therefore, they always acquire either one skill or none. For this reason, I consider a choice between zero and two skills only for those individuals who may at some time find it desirable to acquire two skills.

- $\hat{v}_{20}$ and $\hat{v}_{02}$ converge as g increases and σ decreases but diverge when the reverse is true:

$$\frac{\partial \hat{v}_{20}}{\partial g} > 0,\ \frac{\partial \hat{v}_{02}}{\partial g} < 0,\ \frac{\partial \hat{v}_{20}}{\partial \sigma} < 0,\ \frac{\partial \hat{v}_{02}}{\partial \sigma} > 0$$

In consequence, the fraction of individuals who prefer one skill to none is larger when general ability is higher and perceived uncertainty is lower. Similarly, the fraction of individuals who prefer two skills to one is larger when g is lower and σ is higher, while the fraction of individuals who prefer two skills to none, given that they may at some time prefer two skills to one, is larger when g is higher and σ is lower.

In particular, the marginal surplus from the level of a second skill decreases in g and increases in σ. For this reason, two skills are less likely to be attractive, relative to one, when g is high and σ is low.

The relative positions of these five cutoffs are of interest in determining the overall skill acquisition pattern across individuals. Moreover, since g is normalized between 0 and 1, these relative positions depend primarily on the value of σ. At extremely low uncertainty ($\sigma \rightarrow 0$), it follows that

$$0 = \hat{v}_{01} < \hat{v}_{20} = \hat{v}_{02} < \hat{v}_{12} = \hat{v}_{21} < 1$$

In this case, all individuals find it worthwhile to acquire one skill, though none face sufficient incentive to acquire two. The absolute value of the negative marginal surplus from level investment in a second skill is particularly large for low uncertainty, since individuals optimally invest in a higher level of skill when uncertainty is low, and it thus swamps the potential gain that workers would derive from a greater breadth of competence. Therefore, they all opt for full specialization.

In contrast, at extremely high uncertainty ($\sigma \rightarrow \infty$), it follows that

$$0 < \hat{v}_{20} < \hat{v}_{12} < \hat{v}_{02} < \hat{v}_{21} = \hat{v}_{01} = 1$$

Therefore, at the highest levels of uncertainty, individuals of high specific-ability balance acquire no skills, while those of lowest specific-ability balance all acquire two. Those who do choose to invest in human capital prefer two skills to none but would find investment in any one skill insufficiently rewarding. Acquiring one skill at such high uncertainty is costly but provides no benefit. In fact, no individual acquires exactly one skill in this case.

A variety of intermediate skill acquisition patterns exist relative to these two limiting cases. In particular, simulations indicate that the skill acquisition pattern, from individuals of highest to lowest specific-ability balance, generally does proceed from zero skills, to one, to two, for intermediate values of uncertainty. These simulation results are presented in Appendix II.

Thus, the primary qualitative predictions of the basic model in the case of high perceived uncertainty[4] may be summarized as follows:

- Individuals of greater specific-ability balance tend to acquire fewer skills.
- Individuals of higher general ability are more likely to acquire a positive number of skills but are also more likely to specialize fully.
- *Therefore, those individuals who are observed to acquire multiple skills should be those who are less balanced and less generally able.*

3.1.2 Extended model

As the results are qualitatively similar for both the economy-of-scope and learning-by-doing, I discuss only the first of these cases. The purpose of the extended analysis is to demonstrate that the most balanced individuals do find it optimal to acquire two skills in some cases. This result contrasts with that of the basic model, which suggests that the most balanced individuals never acquire two skills. I therefore restrict attention only to the set of most balanced individuals.

Despite this restriction, the skill number choice results derived for the extended model are considerably more general than those for the basic model. In particular, the effects of general

[4]I emphasize the results for high perceived uncertainty, because this case is currently likely to be most relevant empirically. As noted earlier, empirical evidence suggests not only that wage dispersion has recently increased among skilled workers, but also that at least some workers view this increased wage uncertainty as a reason to diversify their skills.

ability, specific-ability balance, and uncertainty on the number of skills acquired are no longer monotonic.

Moreover, note that the analogous set of the "most balanced" individuals for the economy-of-scope now comprises those with $v < \frac{1}{2\pi\gamma}$, rather than simply those with $v < \frac{1}{2\pi}$. However, the latter case is subsumed in the former, given that $\gamma < 1$ by definition. Thus, the results for this case are potentially relevant not only for those with $v < \frac{1}{2\pi}$, but for all $v \in (0, 1)$, depending on the value of γ.

In the extended model, as in the basic model, it is appropriate to define indifference cutoffs that indicate the regions in which individuals prefer a specific number of skills. In particular, the cutoff $\hat{v}_{01}$ from the basic model remains relevant, since the presence of a cost complementarity only influences individual choices concerning the acquisition of two skills. I define the following two additional cutoffs for the economy-of-scope:

- $\hat{v}_{21}^A$, where $0 \leq \hat{v}_{21}^A \leq \frac{1}{2\pi\gamma}$:

  Individuals with $v < \frac{1}{2\pi\gamma}$ prefer two skills to one if $v < \hat{v}_{21}^A$. Otherwise, they prefer one skill to two.

- $\hat{v}_{20}^A$, where $0 \leq \hat{v}_{20}^A \leq \frac{1}{2\pi\gamma}$:

  Individuals with $v < \frac{1}{2\pi\gamma}$ prefer two skills to none if $v < \hat{v}_{20}^A$. Otherwise, they prefer zero skills to two.

The first of these cutoffs is relevant for $\gamma > \frac{1}{2}$ and sufficiently high uncertainty. More specifically, all of the most balanced individuals prefer two skills to one if the cost complementarity is sufficiently great $\left(\gamma \leq \frac{1}{2}\right)$. Intuitively, the marginal surplus from the second skill is always positive for the most balanced individuals in this case, in contrast with that of the basic model.

In addition, for $\gamma > \frac{1}{2}$, if uncertainty is low, then all of the most balanced individuals prefer one skill to two, as they face insufficient incentive to diversify. The marginal surplus from level in this case is large and negative, dominating the marginal surplus from diversity when it is positive.

However, those individuals with $v < \frac{1}{2\pi\gamma}$ prefer two skills to one under high uncertainty in this case. For these individuals, the marginal surplus from diversity is positive, and that from level is negative but sufficiently small in absolute value not to deter investment. Finally, as $\gamma \to 1$, which was considered for the basic model, no value of σ induces these individuals to acquire two skills, as the marginal cost is simply too large in the absence of a cost complementarity.

The second of these cutoffs is relevant for all values of $\gamma \in (0, 1)$. However, if $\gamma > \frac{1}{2}$, then uncertainty must be sufficiently low in order for a positive value of this cutoff to exist. That is, if the cost complementarity is low and uncertainty is high, then none of the most balanced individuals prefer two skills to zero, because the absolute cost per skill is too high relative to the benefit from two. In contrast, when uncertainty is low, two skills become attractive relative to none.

More explicitly, the behavior of these cutoffs, given that they exist, depends on g, σ, and γ as follows:

- $\hat{v}^A_{21}$ decreases in g and γ and increases in σ:

$$\frac{\partial \hat{v}^A_{21}}{\partial g} < 0, \ \frac{\partial \hat{v}^A_{21}}{\partial \sigma} > 0, \ \frac{\partial \hat{v}^A_{21}}{\partial \gamma} < 0$$

- $\hat{v}^A_{20}$ increases in g and decreases in σ and γ:

$$\frac{\partial \hat{v}^A_{20}}{\partial g} > 0, \ \frac{\partial \hat{v}^A_{20}}{\partial \sigma} < 0, \ \frac{\partial \hat{v}^A_{20}}{\partial \gamma} < 0$$

As before, it is possible to examine the limiting behavior of these cutoffs under different extremes of perceived uncertainty. However, the skill acquisition pattern of the most balanced individuals also now depends on the magnitude of the cost complementarity. For $\gamma < \frac{1}{2}$, all of the most balanced individuals prefer two skills to one or zero, regardless of the level of perceived uncertainty.[5] In contrast, when $\gamma \geq \frac{1}{2}$, none of the most balanced individuals

[5]This result should be interpreted with caution. For the economy-of-scope discussed here, very low γ will imply, for some individuals, that the *total* cost of acquiring two skills is less than that of acquiring one. However, the results of this subsection are qualitatively similar to those for the case of learning-by-doing, so the qualitative implications provided here are representative.

acquire two skills. Under low uncertainty, they face insufficient incentive to diversify, while, under very high uncertainty, the marginal benefit from skill level is simply too small relative to the cost of the second skill. These results are consistent with those derived from the basic model, in that none of the most balanced individuals acquire two skills as γ approaches 1. However, it is clear that alternative cases do exist when γ falls sufficiently.

More generally, the intuition underlying skill investment patterns for all individuals is as follows. The least balanced are the most likely to face a positive marginal surplus from diversity, because a very unbalanced individual acquiring his most-preferred skill faces a lower cost per skill than would a more balanced individual acquiring that same skill. In the presence of a significant cost complementarity, the set of those with a positive marginal surplus from diversity expands to include the most balanced individuals, because the marginal surplus from diversity is not linear in v. Those of intermediate specific-ability balance follow as the cost complementarity increases. Furthermore, all individuals generally face a negative marginal surplus from level in the absence of a significant cost complementarity. However, as the cost complementarity increases, this marginal surplus becomes positive for all individuals. The absolute value of the marginal level surplus term increases with general ability. Therefore, more generally able individuals are more likely to prefer a second skill under a significant cost complementarity and less likely to prefer a second skill in the absence of one, relative to their peers in specific-ability balance, and for a given level of perceived uncertainty. Finally, individuals of greater specific-ability balance will tend to prefer a greater number of skills than before as the cost per skill falls, so it follows simply that the presence of a significant cost complementarity is, in fact, sufficient to induce a greater fraction of individuals to acquire multiple skills under high uncertainty.

As a supplement to the analytic results discussed above, simulations can be used to demonstrate that this intuition holds for the entire spectrum of individual profiles. A variety of such simulations for this case also are provided in Appendix II.

In summary, the extended model implies the following more general results for the case of high perceived uncertainty:

- Individuals of lesser specific-ability balance are most likely to acquire multiple skills. However, those of greater specific-ability balance may be more likely than those of intermediate specific-ability balance to acquire multiple skills, depending on the extent to which the cost per skill falls as the number of skills acquired increases.

- Individuals of higher general ability are more likely to acquire a positive number of skills but may be either more or less likely to specialize fully, depending on the extent to which the cost per skill falls as the number of skills acquired increases.

- *Therefore, those individuals who are observed to acquire multiple skills should be those who are either most or least balanced and either most or least generally able. Moreover, an observed positive relation between either specific-ability balance or general ability and skill number would suggest that the cost per skill falls as the number of skills acquired increases.*

3.2 Secondary analytic results – skill level and diversity

Several additional implications obtain with regard to the optimal choices of skill level and diversity. First, for both the basic and extended models, the optimal level of skill acquired increases in general ability and decreases in uncertainty:

$$\frac{\partial \lambda_{ij}^*}{\partial g} > 0, \ \frac{\partial \lambda_{ij}^*}{\partial \sigma} < 0,$$

for all $i = 1, 2$ and $j = 1, 2$. Second, also in both models, if two skills are acquired optimally, the diversity of those skills increases in specific-ability balance:

$$\frac{\partial \delta^*}{\partial v} < 0,$$

for those individuals who do not find maximum diversity (π) optimal. Finally, in the case of the extended model, optimal skill level and diversity both increase in the cost complementarity:

$$\frac{\partial \lambda_{2j}^*}{\partial \gamma} < 0, \; \frac{\partial \delta^*}{\partial \gamma} < 0,$$

for $j = 1, 2$.

Thus, as should be expected, more generally able individuals optimally acquire a higher level of skill; more balanced individuals optimally acquire a greater diversity of skills if they acquire more than one; and the level and diversity of skills acquired tends to be greater when a marginal skill is less costly. These results come primarily from the assumptions that I originally made about the functional form of the benefit function.

Figure 3.1: Ability cutoffs and preferences for the basic model

Preferences for one skill vs. zero skills

most balanced   least balanced

0 -------------- $\hat{v}_{01}$ -------------------------- 1
$\lfloor$ $0 \succ 1$ $\rfloor\lfloor$ $1 \succ 0$ $\rfloor$

Preferences for two skills vs. one skill

most balanced   least balanced

0 ------- $\frac{1}{2\pi}$ ------------------- $\hat{v}_{12}$ ------ $\hat{v}_{21}$ ----- 1
$\lfloor$ $1 \succ 2$ $\rfloor\lfloor$ $1 \succ 2$ $\rfloor\lfloor$ $2 \succ 1$ $\rfloor\lfloor$ $1 \succ 2$ $\rfloor$

Preferences for two skills vs. zero skills for $v > \frac{1}{2\pi}$

most balanced   least balanced

0 ---- $\frac{1}{2\pi}$ ---- $\hat{v}_{20}$ ------------------ $\hat{v}_{02}$ ----- 1
$\lfloor$ $2 \succ 0$ $\rfloor\lfloor$ $0 \succ 2$ $\rfloor\lfloor$ $2 \succ 0$ $\rfloor$

Chapter 4

Data and Empirical Framework

4.1 Description of the Data

4.1.1 Overview

The full data sample comprises student-level information for 17,099 individuals[1] who graduated from UNC-Chapel Hill with a bachelor's degree between 1999 and 2006. About half of these individuals were born during or after 1980, though birth years in the sample span the period from 1975 to 1985. The mean age at graduation was 22. Most of the individuals (81%) were residents of North Carolina when they began attending UNC-Chapel Hill. Women comprise about 63% of the sample, and 81% of the sample is White. Blacks and Asians make up 11% and 5% of the sample, respectively. The graduates are relatively evenly distributed across graduation years, except for 1999, which is the first year for which any data are available.

In addition to demographic information, such as race, gender, age, and residency status, complete transcript data are available for these individuals. Specifically, the data contain information about all courses taken and grades received, as well as major field declarations, for each semester of enrollment at UNC-Chapel Hill. On average, the individuals in the sample took fewer than 3 courses in any given department but took a total of 38 courses for graduation.

[1] I exclude individuals who were missing SAT scores, were classified as graduate students, transferred to UNC after their freshman year, or were more than 25 years of age at graduation. These restrictions remove cases where ability is not known and allow focus to be placed on 'traditional' undergraduates. These exclusions reduce the sample size from 25,497 to 17,099.

Of these 38 courses, approximately 17 fell within students' major fields.[2] In addition, these students placed out of 8 courses, took 31 courses below the 100-level[3], and took 6 courses at or above the 100-level on average.

Unfortunately, income, tuition, and family background information is not available for the majority of these individuals. However, an internal survey was administered to the classes of 2003-2006 assessing four items: whether or not financial aid was received, joint parental income, paternal educational attainment, and maternal educational attainment. Complete unit responses to the survey are available for 4,540 individuals, or 27% of the sample.

Of those students who completed the survey, 63% said that they had received financial aid. A majority of the survey completers reported that their fathers had earned graduate degrees (39%) and that their mothers had completed college (42%) or graduate school (28%). A third of students reported parental income in the range of \$30,000-\$75,000, while approximately 20% of students reported parental incomes in each of the three higher income categories: between \$75,000 and \$100,000, between \$100,000 and \$150,000, and above \$150,000.

More specifically, the data include the following primary variables:

- **Two Majors**: A dummy variable indicating the number of distinct majors declared during the last semester of enrollment prior to graduation. Equal to 1 if two majors and 0 otherwise.
- **Initial Residency Status**: dummy variable indicating residency during the first semester of enrollment. Equal to 1 if resident and 0 otherwise.
- **Year of Birth**: The student's year of birth.
- **White**, **Asian**, **Black**, **Other**: dummy variables indicating ethnicity. Each is equal to 1 if the individual is of the corresponding ethnicity and 0 otherwise.

[2]The number of courses and fraction within major fields have remained relatively stable during the time period considered, so it is unlikely that any changes in course requirements have contributed to the recent increase in double majors.

[3]All undergraduate courses at UNC prior to the curriculum change of 2006 were categorized at the 100-level or below. Courses at the 200-level or above were graduate level.

- **Ability Balance**: the absolute value of the difference between SAT verbal and math scores. Smaller values indicate greater specific-ability balance.

- **Normalized Ability Balance**: a normalized version of the ability balance measure that is scaled based on the full sample to fall between 0 and 1.

- **General Ability**: the average of SAT verbal and math scores. Larger values indicate greater general ability.

- **Normalized General Ability** a normalized version of the general ability measure that is scaled based on the full sample to fall between 0 and 1.

- **Male**: dummy variable indicating gender. Equal to 1 if male and 0 otherwise.

- **Grad Year**: A class variable indicating the year of graduation.

- **Grade Point Average**: The student's GPA at graduation.

- **Total Courses Taken**: The total number of courses taken at UNC in which the student received a standard course grade greater than F. (Standard grades include A+, A, A-, B+, B, etc.)

- **Courses Required for Majors**: The total minimum number of core courses required by the major field departments for the major or majors selected. This measure takes into account any overlap in required courses when two majors are chosen.

- **Courses Placed Out**: The number of courses exempted by either advanced placement or examination.

- **Transcript Index**: The average number of courses taken by the student in any given department.[4]

- **Upper Level Courses**: The number of courses taken at the 100-level or above.

[4]The number of departments in which courses were taken and the transcript index have a highly significant correlation of -.70, regardless of whether the full sample or the survey sample is considered. Therefore, the transcript index tends to be higher when the number of departments in which courses are taken is smaller. The remaining variation derives from the fact that students graduate with different numbers of courses overall.

- **Lower Level Courses**: The number of courses taken below the 100-level.
- **Received Financial Aid**: A survey dummy variable indicating whether the student received financial aid.
- **Parental Income**: A categorical survey variable indicating joint parental income.
- **Father's Education**: A categorical survey variable indicating the educational attainment of the father.
- **Mother's Education**: A categorical survey variable indicating the educational attainment of the mother.

Summary statistics for the numeric variables, for both the full sample and for survey completers, and for both genders, are presented in Tables 4.1-4.6 below. Table 4.7 presents a test of means for the full sample versus the set of survey completers. In addition, Table 4.8 presents unweighted frequencies for the survey variables for the complete survey cases.

As can be seen from these tables, important differences exist in the characteristics of survey completers relative to those of the full sample. Specifically, survey participants are more likely to have been residents upon matriculation and to have graduated recently (by virtue of when the survey was administered). In addition, survey respondents were nearly a year younger at graduation, placed out of more courses, exhibited higher measured general ability and ability balance, graduated with higher GPAs, and were more likely to have a double major.

In addition, student characteristics vary systematically by gender, and in the same directions, within each of these two samples. On average, men have higher general ability but lower ability balance and a lower GPA than women. In addition, the men are more likely to be White and tend to place out of more courses and take more upper level courses than the women, though both groups take about the same number of courses overall and achieve similar degrees of specialization as measured by the transcript index. Men in the survey sample are slightly more likely than women to double major, but the overall fraction of double majors is close across genders.

Table 4.9 presents the correlations among the four survey variables. Receipt of financial aid is inversely related to the other family background measures. Income and parental education move in the same direction, as expected, and this relationship is strongest for the father's education.

Finally, Table 4.10 presents a summary of how the normalized ability measures and the fraction of students with two majors has evolved over time at UNC. Normalized general ability and the proportion with two majors have increased together from graduating class to graduation class, from 0.54 to 0.67 and from 20% to 28%, respectively. However, normalized ability balance has remained relatively stable, at around 0.15.

Table 4.1: Summary Statistics for All Students (N=17,099)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.81	0.40	0	1
Year of Birth	1981	2.11	1975	1985
Age at Grad	22.33	0.67	19	25
White	0.81	0.40	0	1
Asian	0.05	0.22	0	1
Black	0.11	0.31	0	1
Other	0.03	0.18	0	1
Ability Balance	58.63	46.03	0	370
Normalized Ability Balance	0.15	0.12	0	0.97
General Ability	620.75	70.33	330	800
Normalized General Ability	0.63	0.14	0.04	1
GPA	3.11	0.46	1.84	4
Male	0.37	0.48	0	1
Grad Year	2003	2.02	1999	2006
Two Majors	0.24	0.43	0	1
Total Courses Taken	37.55	4.14	17	65
Transcript Index	2.32	0.35	1.21	4.89
Courses Required for Majors	17.40	7.39	8	54
Courses Placed Out	7.54	6.95	0	49
Upper Level Courses	6.37	4.58	0	30
Lower Level Courses	31.16	5.63	8	63

Table 4.2: Summary Statistics for All Male Students (N=6,375)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.79	0.41	0	1
Year of Birth	1881	2.15	1975	1985
Age at Grad	22.44	0.71	19	25
White	0.83	0.37	0	1
Asian	0.06	0.23	0	1
Black	0.08	0.27	0	1
Other	0.03	0.18	0	1
Ability Balance	62.7	48.57	0	340
Normalized Ability Balance	0.17	0.13	0	0.89
General Ability	636.42	71.46	330	800
Normalized General Ability	0.67	0.15	0.04	1
GPA	3.06	0.49	1.84	4
Grad Year	2003	2.03	1999	2006
Two Majors	0.24	0.43	0	1
Total Courses Taken	37.58	4.06	19	61
Transcript Index	2.34	0.36	1.39	4.56
Courses Required for Majors	17.36	7.70	8	54
Courses Placed Out	8.56	7.53	0	49
Upper Level Courses	7.49	5.02	0	28
Lower Level Courses	30.08	5.77	10	61

Table 4.3: Summary Statistics for All Female Students (N=10,724)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.82	0.38	0	1
Year of Birth	1981	2.09	1975	1985
Age at Grad	22.26	0.64	19	25
White	0.79	0.41	0	1
Asian	0.05	0.22	0	1
Black	0.13	0.33	0	1
Other	0.03	0.18	0	1
Ability Balance	56.21	44.29	0	370
Normalized Ability Balance	0.15	0.12	0	0.97
General Ability	611.43	67.96	350	800
Normalized General Ability	0.62	0.14	0.08	1
GPA	3.14	0.44	1.86	4
Grad Year	2003	2.02	1999	2006
Two Majors	0.24	0.43	0	1
Total Courses Taken	37.52	4.19	17	65
Transcript Index	2.32	0.34	1.21	4.89
Courses Required for Majors	17.42	7.20	8	46
Courses Placed Out	6.93	6.50	0	47
Upper Level Courses	5.70	4.16	0	30
Lower Level Courses	31.81	5.44	8	63

Table 4.4: Summary Statistics for Survey Completers (N=4,540)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.84	0.37	0	1
Year of Birth	1983	0.91	1979	1985
Age at Grad	22.24	0.58	19	25
White	0.79	0.41	0	1
Asian	0.06	0.24	0	1
Black	0.10	0.30	0	1
Other	0.05	0.21	0	1
Ability Balance	56.71	44.87	0	370
Normalized Ability Balance	0.15	0.12	0	0.97
General Ability	634.90	65.58	410	800
Normalized General Ability	0.66	0.13	0.20	1
GPA	3.20	0.43	1.93	4
Male	0.36	0.48	0	1
Grad Year	2005	0.79	2002	2006
Two Majors	0.28	0.45	0	1
Total Courses Taken	36.45	4.27	19	56
Transcript Index	2.31	0.36	1.29	4.20
Courses Required for Majors	17.95	7.40	8	42
Courses Placed Out	9.36	7.60	0	47
Upper Level Courses	6.08	4.64	0	28
Lower Level Courses	30.35	5.65	11	56

Table 4.5: Summary Statistics for Male Survey Completers (N=1,623)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.81	0.39	0	1
Year of Birth	1983	0.93	1979	1985
Age at Grad	22.32	0.60	19	25
White	0.83	0.38	0	1
Asian	0.07	0.25	0	1
Black	0.06	0.24	0	1
Other	0.05	0.21	0	1
Ability Balance	60.22	45.77	0	290
Normalized Ability Balance	0.16	0.12	0	0.76
General Ability	653.08	65.02	410	800
Normalized General Ability	0.70	0.13	0.20	1
GPA	3.18	0.44	1.97	4
Grad Year	2005	0.78	2002	2006
Two Majors	0.29	0.45	0	1
Total Courses Taken	36.30	4.29	19	51
Transcript Index	2.33	0.37	1.39	4.2
Courses Required for Majors	17.96	7.8	8	42
Courses Placed Out	10.75	8.16	0	42
Upper Level Courses	7.26	5.23	0	28
Lower Level Courses	29.03	5.86	11	46

Table 4.6: Summary Statistics for Female Survey Completers (N=2,917)

Variable	Mean	Std Dev	Min	Max
Initial Residency Status	0.85	0.36	0	1
Year of Birth	1983	0.90	1980	1985
Age at Grad	22.19	0.56	19	25
White	0.77	0.42	0	1
Asian	0.06	0.23	0	1
Black	0.12	0.33	0	1
Other	0.05	0.22	0	1
Ability Balance	54.75	44.24	0	370
Normalized Ability Balance	0.14	0.12	0	0.97
General Ability	624.79	63.69	425	800
Normalized General Ability	0.64	0.13	0.23	1
GPA	3.21	0.44	1.93	4
Grad Year	2005	0.80	2002	2006
Two Majors	0.28	0.45	0	1
Total Courses Taken	36.53	4.26	19	56
Transcript Index	2.30	0.35	1.29	4.09
Courses Required for Majors	17.95	7.16	8	41
Courses Placed Out	8.59	7.15	0	47
Upper Level Courses	5.42	4.13	0	28
Lower Level Courses	31.09	5.39	11	56

Table 4.7: Comparisons of Means for the Two Samples

Sample	All	Survey[5]
N	17,099	4,540
Initial Residency Status	0.81	0.84*
Year of Birth	1981	1983*
Age at Grad	23.33	22.24*
White	0.81	0.79*
Black	0.11	0.10
Asian	0.05	0.06
Other	0.03	0.05*
Ability Balance	58.63	56.75*
Normalized Ability Balance	0.15	0.15*
General Ability	620.75	634.90*
Normalized General Ability	0.63	0.66*
GPA	3.11	3.20*
Male	0.37	0.36*
Grad Year	2003	2005*
Two Majors	0.24	0.28*
Total Courses Taken	37.55	36.44*
Transcript Index	2.33	2.31*
Courses Required for Majors	17.40	17.95*
Courses Placed Out	7.54	9.36*
Upper Level Courses	6.37	6.08*
Lower Level Courses	31.16	30.35*

Table 4.8: Frequencies for Survey Completers

Variable	Value	Frequency	Percent
Received Financial Aid	Yes	2,857	62.80
	No	1,689	37.20
Parental Income	$30,000 or less	312	6.87
	$30,001-$75,000	1,376	30.31
	$75,001-$100,000	923	20.33
	$100,001-150,000	961	21.17
	more than $150,000	968	21.32
Father's Education	less than HS Graduate	55	1.21
	HS Graduate	454	10.00
	Some College	627	13.81
	College Degree	1,626	35.81
	Graduate Degree	1,778	39.16
Mother's Education	less than HS Graduate	44	0.97
	HS Graduate	458	10.09
	Some College	851	18.74
	College Degree	1,914	42.16
	Graduate Degree	1,273	28.04

Table 4.9: Correlations of Survey Responses (N=4,540)

Variable	Financial Aid	Income	Father's Educ.	Mother's Educ.
Financial Aid	1.00			
Income	-0.37*	1.00		
Father's Education	-0.24*	0.42*	1.00	
Mother's Education	-0.17*	0.32*	0.52*	1.00

* = p-value < 0.0001.

Table 4.10: Mean Normalized Ability Measures and Fraction with Two Majors by Year

	Sample:	All	Survey
Graduation Year			
1999	General Ability	0.54	.
	Ability Balance	0.23	.
	Two Majors	0.20	.
2000	General Ability	0.61	.
	Ability Balance	0.16	.
	Two Majors	0.18	.
2001	General Ability	0.61	.
	Ability Balance	0.15	.
	Two Majors	0.21	.
2002	General Ability	0.62	.
	Ability Balance	0.15	.
	Two Majors	0.23	
2003	General Ability	0.64	0.71
	Ability Balance	0.15	0.15
	Two Majors	0.25	0.26
2004	General Ability	0.64	0.65
	Ability Balance	0.15	0.15
	Two Majors	0.25	0.26
2005	General Ability	0.65	0.66
	Ability Balance	0.15	0.15
	Two Majors	0.27	0.28
2006	General Ability	0.67	0.68
	Ability Balance	0.15	0.15
	Two Majors	0.28	0.29

4.1.2 Majors and the Curriculum

Graduates from UNC-Chapel Hill are allowed to select at most two majors. All students must fulfill a spectrum of General College distribution requirements in the College of Arts and Sciences during their first two years of study. Thereafter, each student chooses either to remain in the College of Arts and Sciences while completing his major requirements or to apply to one of the various professional schools, such as Nursing or Business, to complete his degree. Several additional Perspective requirements must generally be satisfied as upper-level distributional electives outside the major department, in addition to the major coursework.

The type of major that each student attempts, as well as which schools (i.e., Arts and Sciences, Business, Education, etc.) he attends for his major(s) determine the degree that he receives. As should be expected, an individual with two majors that would each normally result in a Bachelor of Arts degree receives a Bachelor of Arts degree. However, an individual with at least one major that would normally lead to a Bachelor of Science degree receives a Bachelor of Science degree, regardless of his second major choice. For example, an individual majoring in Applied Sciences (which confers a B.S.) and English (which confers an A.B.) would receive only a B.S. at graduation.

Approximately 24% of the individuals in the sample graduated with two majors. However, as indicated in Tables 4.11 and 4.12, the fraction of individuals choosing two majors varies widely both by degree type and by the school from which the individual received his degree. Over 29% of those individuals receiving an A.B. chose two majors, while only 10% of those with a B.S. did so.[6] Moreover, most of the double majors are concentrated among individuals who graduated from the College of Arts and Sciences, the School of Business, the School of Education, and the School of Journalism and Mass Communication. These differences may be explained in part by noting that the overlap of a student's distribution requirements and his free electives with his major choice differs widely across majors because of differences in the number of requirements that must be satisfied for each.

[6]The fraction of students obtaining an A.B. has remained relatively stable, at around 72%, during the time period considered. Therefore, it is unlikely that the fact that students with an A.B. are more likely to double major has contributed to the recent increase in double majors.

In general, A.B. majors in the College of Arts and Sciences require between 8 and 15 courses, which can, to varying degrees, be satisfied as part of the distribution requirements. The A.B. degree requirements typically differ from the B.S. requirements for those majors, such as Biology, that offer both tracks in that the A.B. track offers considerably more flexibility in terms of the electives and preparatory course work involved. Degrees from the professional schools of Business, Journalism, and Education also tend to offer considerable flexibility in coursework external to the major.

In contrast, B.S. and B.F.A. majors in the College of Arts and Sciences typically require upwards of twenty courses, though this is compensated for in some cases by a waiver of the upper-level Perspective requirements in other departments. Some of the required courses can be partly satisfied via the distribution requirements, but many of the electives are predetermined, so the overall course load is higher both within and without the major field. In fact, students earning a B.F.A. in art are prohibited by the Department of Art from electing a second major. Degrees from the more science-based professional schools, such as Medicine and Nursing, tend to have similarly demanding course loads.

Table 4.11: Percent Double Majors by Degree Type

Degree Type	No. Obs	Percent with Two Majors
Bachelor of Arts	12,301	29.09
Bachelor of Fine Arts or Music	163	12.27
Bachelor of Science	4,635	10.40

Table 4.12: Percent Double Majors by Degree School

Degree School	No. Obs	Percent with Two Majors
Arts and Sciences	12,110	26.42
Business	1,582	13.08
Dentistry	60	0.00
Education	386	24.87
Information and Library Science	34	29.41
Journalism and Mass Communication	2,100	26.38
Medicine	124	1.61
Nursing	463	0.00
Public Health	240	5.42

4.1.3 Employment Index

As indicated in the theory section of this paper, the similarity of the two majors that an individual would like to acquire should, in part, determine whether he finds it cost-effective to graduate with two majors instead of only one. Since this degree of similarity is observable for those students who had two majors, I create a diversity index that measures skill similarity in terms of the employment profile associated with each major.

Employment information is not available for the UNC graduates in the sample. Therefore, I create the employment diversity index using data from the Baccalaureate and Beyond study of the National Center for Education Statistics (NCES). This study documents the employment experiences of a nationally representative sample of college graduates who received their bachelor's degrees in the early 1990's. In consequence, the NCES sample represents a cohort of college graduates that entered the job market somewhat earlier than the graduates from UNC. Since it is reasonable to think that UNC student perceptions about the job market may have been colored by observing or hearing about the experiences of their predecessors throughout the country, the results of this NCES study provide a rough means of capturing these perceptions.

The raw data from this NCES study cannot be accessed without a special license. However, one 2001 NCES publication contains information about the occupations in which individuals in the study with specific majors ended up a few years after having graduated. I reproduce this information below in Table 4.13, which presents the percentage of individuals with a bachelor's degree in each of 13 major groups who were employed in each of 12 occupational groups in 1997. For example, of those students who majored in mathematics or the physical sciences (major group 11), 26.2% became educators, 16.5% went into business or management, 9% became engineers or architects, 24% went into research or technical positions, and 8.4% went into services. The full set of employment percentages for all 12 occupational groups may be considered the employment profile associated with academic training in math or the physical sciences.

In this context, the similarity of two majors can be characterized roughly as the similarity of the employment profiles associated with the major groups in which those two majors fall. For example, if the employment profiles for business majors and social science majors are very similar, then we may infer that these two areas of study have much in common. This combination of majors would correspond to a low diversity choice.

I measure the similarity of two employment profiles as simply the sum of the absolute differences in the fractions of graduates employed in each occupational category. More formally, suppose that the employment profile associated with major group j, for $j = 1, ..., 13$, is the set $\{p_{ij}\}$, for $i = 1, ...12$, where p_{ij} is the probability that an individual with a major in group j is employed in occupation group i. Then, the similarity of major group j to major group k is given by

$$\delta_E(j,k) = \sum_{i=1}^{12} |p_{ij} - p_{ik}|$$

This index obviously provides a diversity value of 0 for any two majors in the same major group, or for any major with itself. However, larger numbers correspond to greater diversity. For example, the diversity index value associated with a double major in business and a social science can be calculated as

$$\delta_E(2, 12) = 60.2$$

In contrast, the diversity for a double major in business and nursing is

$$\delta_E(2, 5) = 191.4$$

These numbers indicate that students of business and students of nursing have less similar employment profiles than do students of business and students of the social sciences. Therefore, a student with a double major in business and nursing acquires skills that are less similar, from the perspective of employment outcomes, than one with a double major in business and a social science. For the set of major combinations realized in the data, this index takes on 34 distinct values. In addition, for ease of interpretation, I create a normalized version of this index that falls between 0 and 1.[7]

[7] I also consider a second version of the diversity measure that represents the similarity of majors in terms of

the overlap in the courses required for each major. However, the empirical results for that measure are similar to those for the employment index and smaller in magnitude. Therefore, I omit those results.

Table 4.13: NCES B&B Participants by Undergraduate Major and 1997 Occupation

Occupation	Educators	Business or Management	Engineering or Architecture	Computer Science	Medical Professionals	Editor/ Writers/ Performers
Total	12.5	29.3	5.4	4.9	7.0	4.9
Bachelors degree major						
Applied fields						
Education	73.9	7.0	0.0	0.3	2.1	1.0
Business	3.7	55.8	0.8	5.2	0.5	0.9
Engineering/architecture	1.4	7.5	59.7	6.1	1.1	1.0
Computer science	3.7	12.5	12.9	57.9	1.0	0.0
Health/nursing	0.5	2.8	0.0	0.5	96.2	0.0
Health/other	7.3	7.5	0.0	0.9	68.3	0.0
Communications/journalism	4.1	22.8	0.3	2.8	0.6	23.0
Social work/protective services	6.8	10.5	0.0	0.0	2.0	0.0
Academic fields						
Humanities and arts	17.8	23.4	1.0	3.7	2.0	17.0
Biological/interdisciplinary sciences	24.6	14.6	1.5	0.8	15.2	1.7
Mathematics/physical sciences	26.2	11.5	9.0	7.3	0.8	0.6
Social sciences	8.8	31.9	0.3	1.2	3.1	2.3
Other	8.0	32.0	1.2	1.5	3.5	5.6

Note: Taken from Table 6 of the NCES Statistical Analysis Report for February 2001 entitled *From Bachelor's Degree to Work: Major Field of Study and Employment Outcomes of 1992-93 Bachelors Degree Recipients Who Did Not Enroll in Graduate Education by 1997.* Therefore, the table does not include information about those undergraduates who went directly on to graduate school.

Table 4.14: Table 4.13 continued.

Occupation	Human/ Protective Services	Research/ Scientific/ Technical	Administrative or Legal Support	Mechanics/ Laborers	Services	Other
Total	5.9	4.7	5.5	3.9	14.6	1.0
Bachelors degree major						
Applied fields						
Education	1.4	1.2	4.3	2.8	5.2	0.9
Business	2.0	1.8	4.7	3.5	20.3	0.8
Engineering/architecture	0.5	9.0	1.2	7.5	3.9	1.1
Computer science	0.9	3.3	1.3	2.2	3.3	0.8
Health/nursing	0.0	0.0	0.0	0.0	0.0	0.0
Health/other	3.3	3.6	2.7	0.5	6.0	0.0
Communications/journalism	1.9	3.2	5.6	2.5	33.0	0.0
Social work/protective services	59.8	0.9	10.9	1.6	6.7	0.6
Academic fields						
Humanities and arts	4.8	4.0	6.3	3.8	15.0	0.9
Biological/interdisciplinary sciences	1.5	23.6	2.3	5.2	6.2	2.6
Mathematics/physical sciences	1.7	24.0	5.8	4.0	8.4	0.7
Social sciences	16.4	3.5	9.3	3.0	17.7	2.3
Other	8.8	5.0	5.4	12.3	15.2	1.3

Table 4.15: Normalized Employment Index (Observed for Double Majors Only)

Sample	No. Obs	Mean	Std Dev	Min	Max
All	4,082	0.33	0.26	0	1
Survey	1,273	0.32	0.26	0	1
All Men	1,539	0.30	0.25	0	0.94
Survey Men	463	0.32	0.26	0	0.94
All Women	2,543	0.34	0.26	0	1
Survey Women	810	0.33	0.25	0	1

Summary statistics for the normalized employment index, as well as its correlations with the ability measures for those cases with two majors, are presented in the two tables below. General ability and the ability balance measure have a small but statistically significant positive correlation, though breaking these correlations down by gender suggests that this pattern holds only for women. Thus, more generally able women are also more likely to be less balanced in this sample[8], though this is not necessarily true for men. In addition, general ability has a small negative correlation with the normalized employment index for women, while ability balance is not correlated with this diversity measure. None of these measures are correlated for men or for the survey respondents, although men do exhibit a 6% restriction of range in the employment index relative to women. Thus, the most diversified women choose two majors that are less similar than do the most diversified men. Moreover, the mean employment index for women slightly exceeds that for men in both samples, indicating that women with two majors are, on average, acquiring less similar skills than are men with two majors.

[8] Recall that the normalized ability balance measure is inversely related to actual ability balance.

Table 4.16: Correlations of Ability with Diversity (Observed for Double Majors Only)

All	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	0.03*	1.00	
	Norm. Employment Index	-0.05*	0.00	1.00
Survey	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	0.02	1.00	
	Norm. Employment Index	-0.03	0.01	1.00
All Men	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	-0.02	1.00	
	Norm. Employment Index	0.02	0.04	1.00
Survey Men	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	-0.01	1.00	
	Norm. Employment Index	-0.01	0.07	1.00
All Women	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	0.04**	1.00	
	Norm. Employment Index	-0.06**	-0.02	1.00
Survey Women	Variable	General Ability	Ability Balance	Norm. Employment Index
	General Ability	1.00		
	Ability Balance	0.02	1.00	
	Norm. Employment Index	-0.04	-0.05	1.00

* = significance at the 5%-level or below.

4.1.4 What It Means to Double Major

From a purely descriptive perspective, the students who chose two majors differ in several respects from those who chose only one. With regard to demographics, double majors tend to be slightly younger at graduation and are more likely to be White. However, among those students who completed the family background survey, there are no significant differences with respect to the receipt of financial aid or parental income and education.

With regard to academic choices and abilities, double majors tend to be of higher general ability and to be more balanced in their specific abilities. They also graduate with a higher GPA, place out of more courses upon matriculation, take more courses to graduate, and take a higher level of courses on average. These students are also less likely to be North Carolina residents during their first year of enrollment (i.e., they are out-of-state matriculants and therefore faced higher admissions standards).

These observations suggest that the students who graduate with two majors not only begin college better prepared on average but also commit themselves to a more challenging academic program by selecting two majors rather than one. In addition, it is noteworthy that the transcript index (which is the average number of courses taken in any given department) for students with two majors is slightly higher on average than that for those with one major, suggesting that double majors actually achieve a greater average degree of specialization by choosing more than one field of study. Thus, from the perspective of acquiring a small number of distinct, marketable skills (represented by majors) at the level of a college degree, the choice of a second major represents a diversification decision. However, from the perspective of the wide range of coursework taken at varying levels and in various departments during the course of an undergraduate career, the selection of a second major is actually a decision to be more specialized.

Table 4.17: Two-sided Test of Means: One vs. Two Majors

All	Variable	One Major (N=13,017)	Two Majors (N=4,082)
	GPA*	3.07	3.23
	General Ability*	615.28	638.18
	Normalized General Ability*	0.62	0.70
	Ability Balance*	59.10	57.15
	Normalized Ability Balance*	0.16	0.15
	Total Courses Taken*	37.38	38.10
	Upper Level Courses*	6.19	6.95
	Lower Level Courses	31.17	31.12
	Courses Placed Out*	6.92	9.52
	Courses Required for Majors*	16.13	21.42
	Transcript Index*	2.28	2.47
	Age at Graduation*	22.34	22.28
Survey	Variable	One Major (N=3,267)	Two Majors (N=1,273)
	GPA*	3.17	3.29
	General Ability*	629.40	649.03
	Normalized General Ability*	0.65	0.69
	Ability Balance	57.11	55.66
	Normalized Ability Balance	0.15	0.15
	Total Courses Taken*	36.22	37.03
	Upper Level Courses*	5.87	6.62
	Lower Level Courses	30.34	30.39
	Courses Placed Out*	8.62	11.25
	Courses Required for Majors*	16.60	21.44
	Transcript Index*	2.27	2.44
	Age at Graduation	22.24	22.22

*=significant difference at the 5% level or below.

Table 4.18: Two-sided Test of Proportions: One vs. Two Majors

All	Variable	One Major (N=13,017)	Two Majors (4,082)
	Resident*	0.82	0.77
	Male	0.37	0.37
	White*	0.80	0.81
	Asian	0.05	0.05
	Black*	0.11	0.09
	Other*	0.03	0.04
Survey	Variable	One Major (N=3,267)	Two Majors (N=1,273)
	Resident*	0.85	0.79
	Male	0.35	0.36
	White	0.79	0.80
	Asian	0.06	0.05
	Black	0.10	0.09
	Other	0.05	0.05

*=significant difference at the 5% level or below.

Table 4.19: Two-sided Test of Survey Proportions: One vs. Two Majors

Variable	Value	One Major	Two Majors
Received Financial Aid	Yes	0.63	0.62
Parental Income	$30,000 or less	0.07	0.07
	$30,001-$75,000	0.31	0.29
	$75,001-$100,000	0.21	0.20
	$100,001-150,000	0.21	0.22
	more than $150,000	0.21	0.22
Father's Education	less than HS Graduate	0.01	0.01
	HS Graduate	0.10	0.10
	Some College	0.14	0.14
	College Degree	0.36	0.34
	Graduate Degree	0.38	0.41
Mother's Education	less than HS Graduate	0.01	0.01
	HS Graduate	0.10	0.09
	Some College	0.19	0.18
	College Degree	0.42	0.42
	Graduate Degree	0.28	0.29

Note: None statistically significant at conventional levels.

4.2 Estimation Methods

In the empirical portion of this paper, I assess the qualitative relevance of the theory to the case of UNC-Chapel Hill undergraduate degree recipients. In particular, I am interested in the empirical signs and the significances of the effects of general ability, ability balance, and perceived uncertainty, or cohort effects, on the skill investment patterns of these students as a means of determining which students are most and least likely to choose two skills, and which students are likely to be the most or least specialized in their skill choices.

In this context, all skills (i.e., majors) are acquired at the same level, and so the two outcome variables of interest are the number of skills (one or two majors) and their degree of similarity (diversity). Moreover, in this setup, skills are viewed as interesting and useful to the individual only to the extent that they are marketable because an investment of a certain magnitude has been made in those skills. Thus, even though course work taken outside of specific major fields contributes to the overall course load required for graduation, this course work is simply considered noise in the individual's acquired skill set.

However, before proceeding to this analysis, it is instructive to consider the specialization/diversification decision from a more traditional viewpoint as a means of providing context. Many economists have historically viewed this decision as a simple trade-off between depth and breadth. In other words, devoting resources to obtain a greater depth of skill in any one field necessarily reduces the resources that can be devoted to gaining expertise in the other available fields.

In this traditional context, it is sufficient to consider as the outcome variable of interest a single summary index that reflects the individual's degree of specialization. For example, Lazear (2005) uses transcript data for Stanford MBA graduates to examine the relationship between specialization in course work and the subsequent likelihood of becoming an entrepreneur. For this purpose, he defines a specialization measure as the difference between the number of courses taken in the selected MBA field of concentration and the average number of courses taken in any of the other available concentrations. Individuals with higher values on this index are relatively more specialized than their peers.

Analogously, for the case of UNC undergraduates, we can consider the transcript index discussed above. Recall that this index is the average number of courses taken in any given academic department. Unlike Lazear's specialization measure, the transcript index does not evaluate specialization relative to the amount of course work done in a specific concentration. However, this measure is better suited to the UNC undergraduate curriculum, both because students can choose more than one major field and because majors themselves do not correspond uniformly to academic departments. Some majors span several departments, while others fall within only one. As a result, the transcript index summarizes the average degree of student focus in any given department, regardless of how many majors that student has or whether the major is interdisciplinary. Using this transcript index as the (continuous) dependent variable, I carry out a simple ordinary least squares regression to assess the impact of the ability measures and other student characteristics on academic specialization.

After carrying out this introductory analysis, I then turn to the question of what determines skill number and diversity, implicitly holding level constant. To answer this question, I estimate a system of equations, one equation for each of these outcome variables. The first of these equations is an OLS regression of diversity on student characteristics. Diversity is a continuous variable and is measured as the normalized employment index. However, diversity is only observed for those students who have two majors. Therefore, the second equation is a selection equation for which the dependent variable is nominally an indicator of whether the student had a double major. As is common for the analysis of dichotomous dependent variables, I assume that there exists an underlying latent continuous variable that determines the number of majors and that the indicator variable takes on a value of 1 only when this latent variable exceeds a certain threshold.

More formally, it is possible to elaborate this framework as a case of the Heckman selection model. Omitting observation-level subscripts, let δ denote diversity, α_i denote the coefficient for regressor x_i, and ϵ denote a random error term. Then the outcome equation is given by

$$\delta = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_n x_n + \epsilon$$

Moreover, δ is observed if and only if the individual graduates with two majors. Let n be defined as the indicator of a double major:

$$n = \begin{cases} 1 & \text{if } n^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

where n^* is the latent variable that represents an individual's propensity to acquire more skills at the undergraduate level. Then, the selection equation is given by

$$n^* = \beta_0 + \beta_1 z_1 + \beta_2 z_2 + ... + \beta_m z_m + \phi$$

where β_j is the coefficient for regressor z_j and ϕ is a random error term.

As with the OLS regression for the transcript index, the regressors for the outcome equation and the selection equation comprise student-level demographics and academic characteristics. In the absence of a suitable exclusion restriction, I assume that the model is identified solely based on functional form. I estimate this system of equations via maximum likelihood.

Chapter 5

Empirical Results

5.1 Determinants of the Transcript Index

The estimation results for the transcript index indicate that general ability is an important predictor of course-level undergraduate specialization for both men and women at UNC, but that ability balance influences specialization only for women. Moreover, the results suggest that general ability is relatively more important for men than for women and that ability balance plays a larger role for women than does general ability.

The two tables below present ordinary least squares regression specifications predicting the transcript index. Men and women appear to have consistently different skill investment patterns, so I break each specification down by gender. For the full sample, the graduation years 1999 and 2000 are grouped together as the omitted category to avoid collinearity. Similarly, for the survey sample, the combination of years 2002-2004 is omitted. Birth year also tends to be highly collinear with year of graduation, so I present results that control only for the latter of these two variables. Year of graduation is of greater theoretical interest because it may be interpreted as a rough measure of perceived labor market conditions. Finally, I consider only two of the survey variables, namely receipt of financial aid and parental income, as the latter tends to be most highly correlated with parental educational attainment.

The first table below (Table 5.1) presents the results for the sample of all students and a comparable specification for the set of students who responded to the survey. The second table (Table 5.2) considers only the survey respondents and incorporates the two survey variables of interest.

For the full sample, the marginal effect of general ability for men ranges between -.98 courses and 1.02 courses (as general ability varies between 0 and 1), while that for women ranges between -.32 courses and 0.28 courses. These marginal effects are zero when general ability is equal to 0.49 for men and 0.53 for women. As a result, marginal increases in general ability result in increasing marginal specialization above the midpoint of the ability distribution but in decreasing marginal diversification below that midpoint. Therefore, the most and least generally able individuals are the most specialized in terms of the average number of courses taken in any given department, while those in the middle of the general ability distribution take the most varied course load.

For the set of survey completers, the overall specialization pattern is similar, though these effects are moderated by both the restriction of general ability range that we observe for survey completers and by the additional variables that are available from the survey. In Table 5.1, the marginal effect of general ability for men in the survey sample is somewhat smaller and ranges between 0 and 1.72 courses. Thus, all the men in the survey sample are increasingly specialized as their general ability increases. In contrast, the pattern for women is more comparable to, though greater in magnitude than, that for the case of all female students. In this case, the effect of a marginal increase in general ability for female survey respondents ranges between -0.76 courses and 0.42 courses, and is equal to zero when general ability is 0.64. When the survey variables are added to the analysis, as presented in Table 5.2, the effect of general ability persists for men but becomes marginally significant or insignificant for women. In particular, controlling for parental income eliminates the effect of general ability on the transcript index for women.

The effect of ability balance on specialization is insignificant for males in all five specifications. However, the effect of ability balance for females is highly significant and is robust to the inclusion of survey variables in the analysis. Specifically, for the set of all female students, the effect of a marginal decrease in ability balance ranges between -0.16 courses and 0.60 courses, and is equal to zero when the ability balance measure is equal to 0.21. Therefore, for women in the upper quintile of the ability balance distribution (i.e., the most balanced individuals, for whom the balance measure is less than 0.21), a marginal decrease in ability balance leads to decreasing marginal diversification. For women in the lower four quintiles of the ability balance distribution, the reverse is true: a marginal decrease in ability balance leads to increasing marginal specialization. In short, it follows that the most and least balanced women are the most specialized with regard to the average number of courses taken in any given department, while those at the cusp of the highest quintile take the most varied course load. The pattern is similar when the survey sample and the survey variables are considered, though the marginal effect of a decrease in balance varies between -0.29 courses and 0.95 courses, with a value of zero courses when ability balance is approximately 0.23.

As the financial costs of attendance are also of interest as a determinant of skill investment patterns, it is noteworthy that financial aid is a significant predictor of specialization only for women. The effect is small relative to those of the ability measures, however: receipt of financial aid increases specialiation by 0.04 courses for women. Parental income does not appear to be an important predictor of specialization for survey respondents, with the exception of males in the highest income bracket, who tend to be less specialized by 0.09 courses relative to their peers in the lowest income bracket.

The year of graduation is also a more important predictor of specialization for women than for men, as women who graduated during the period 2002-2004 were slightly more specialized than those who graduated in 1999 or 2000, though there is no significant difference in the two time periods for men. In addition, both men and women who graduated in 2006 were considerably less specialized than earlier graduates, usually by at least one-tenth of a course.

Table 5.1: OLS Regression, Transcript Index by Gender

Sample	All		Survey	
Gender	Male	Female	Male	Female
N	6,375	10,724	1,623	2,917
R-squared	0.03	0.01	0.05	0.03
Constant	2.49 (0.07)**	2.38 (0.05)**	2.52 (0.19)**	2.61 (0.13)**
Ability Balance	0.08 (0.09)	-0.16 (0.07)**	0.11 (0.20)	-0.29 (0.14)**
Ability Balance Squared	-0.08 (0.18)	0.38 (0.15)**	-0.09 (0.41)	0.62 (0.29)**
General Ability	-0.98 (0.20)**	-0.32 (0.16)**	-0.88 (0.54)	-0.76 (0.39)**
General Ability Squared	1.00 (0.16)**	0.30 (0.13)**	0.88 (0.39)**	0.59 (0.30)**
Race				
White	.	.	.	.
Asian	0.02 (0.02)	0.00 (0.01)	0.01 (0.04)	0.04 (0.03)
Black	0.02 (0.02)	0.04 (0.01)**	0.00 (0.04)	0.04 (0.02)*
Other	-0.00 (0.02)	-0.03 (0.02)	-0.06 (0.04)	-0.04 (0.03)
Resident	0.03 (0.01)**	0.02 (0.01)*	0.03 (0.02)	-0.03 (0.02)
Year of Graduation				
1999-2000	.	.	.	.
2001	0.03 (0.02)*	0.00 (0.01)	.	.
2002	0.02 (0.02)	0.03 (0.01)**	.	.
2003	0.01 (0.02)	0.03 (0.01)**	.	.
2004	0.02 (0.02)	0.03 (0.01)**	.	.
2005	0.03 (0.02)	0.02 (0.01)	0.00 (0.02)	0.00 (0.02)
2006	-0.11 (0.02)**	-0.08 (0.01)**	-0.15 (0.02)**	-0.10 (0.02)**

***=significant at the 10% level; **=significant at the 5% level or below**

Table 5.2: OLS Regression, Transcript Index by Gender for the Survey Sample

Gender	Male	Female	Male	Female	Male	Female
N	1,623	2,917	1,623	2,917	1,623	2,917
R-squared	0.05	0.03	0.06	0.03	0.06	0.03
Constant	2.52 (0.19)**	2.53 (0.13)**	2.56 (0.19)**	2.56 (0.19)**	2.58 (0.19)**	2.53 (0.13)**
Ability Balance	0.11 (0.20)	-0.29 (0.14)**	0.09 (0.20)	-0.28 (0.14)**	0.09 (0.20)	-0.29 (0.14)**
Ability Balance Squared	-0.09 (0.41)	0.63 (0.29)**	-0.04 (0.41)	0.59 (0.29)**	-0.04 (0.41)	0.62 (0.29)**
General Ability	-0.88 (0.55)	-0.60 (0.39)	-0.83 (0.54)	-0.66 (0.39)*	-0.87 (0.55)	-0.54 (0.39)
General Ability Squared	0.88 (0.40)**	0.47 (0.30)	0.85 (0.39)**	0.53 (0.30)*	0.88 (0.40)**	0.43 (0.30)
Race						
White	.	.	.	.	.	.
Asian	0.01 (0.04)	0.04 (0.03)	0.01 (0.04)	0.04 (0.03)	0.01 (0.04)	0.04 (0.03)
Black	0.00 (0.04)	0.03 (0.02)	-0.01 (0.04)	0.03 (0.02)	-0.01 (0.04)	0.03 (0.02)
Other	-0.06 (0.04)	-0.04 (0.03)	-0.06 (0.04)	-0.04 (0.03)	-0.06 (0.04)	-0.04 (0.03)
Resident	0.03 (0.02)	-0.03 (0.02)	0.03 (0.02)	-0.03 (0.02)	0.03 (0.02)	-0.03 (0.02)
Year of Graduation						
1999-2004	.	.	.	.	.	.
2005	-0.00 (0.02)	0.00 (0.02)	-0.00 (0.02)	0.00 (0.02)	-0.00 (0.02)	0.00 (0.02)
2006	-0.15 (0.02)**	-0.10 (0.02)**	-0.15 (0.02)**	-0.10 (0.02)**	-0.15 (0.02)**	-0.10 (0.02)
Received Financial Aid	-0.00 (0.02)	0.04 (0.01)**	.	.	-0.01 (0.02)	0.04 (0.02)**
Parental Income						
Less than $30K	.	.	.	.	.	.
$30K - $75	.	.	-0.06 (0.04)	-0.01 (0.03)	-0.06 (0.04)	-0.01 (0.03)
$75K - $100	.	.	-0.05 (0.04)	-0.06 (0.03)	-0.05 (0.04)	-0.05 (0.03)*
$100K - $150	.	.	-0.04 (0.04)	-0.01 (0.03)	-0.04 (0.04)	0.00 (0.03)
More than $150K	.	.	-0.09 (0.04)**	-0.04 (0.03)	-0.09 (0.04)**	-0.02 (0.03)

***=significant at the 10% level; **=significant at the 5% level or below**

5.2 Employment Index and the Number of Majors

The estimation results for the employment index (skill diversity) and the number of majors suggest that more generally able men and women are both considerably more likely to double major. However, general ability contributes to less similar majors for men and more similar majors for women. Ability balance is not an important determinant of either major diversity or the number of majors in these samples.

The five tables below present estimation results for the normalized employment index and the number of majors, broken down by gender. In predicting these dependent variables, I correct for selection in the diversity equation only when evidence of selection is present. In particular, I employ the Heckman selection model for the group of all male students but use the more simple combination of OLS and probit regressions for the set of all female students and the set of survey completers, as no selection problem is evident in these cases (i.e., ρ is not statistically significant). The first table presents results for all students, and the subsequent four tables consider a comparable specification for the survey completers, followed by specifications incorporating the survey variables.

For the full sample, a marginal increase in general ability increases the normalized employment index by 0.36 points for men but decreases it by 0.13 points for women. These results suggest that more generally able men prefer majors that are less similar, while the reverse is true for more generally able women. For the survey sample, general ability does not significantly influence major similarity for either gender. In contrast, a marginal increase in general ability increases the likelihood of a double major for both men and women by .43-.51 points at the mean, and this effect is highly significant for both genders, regardless of the sample or covariates considered. Therefore, more generally able individuals of both genders are more likely to double major, but the preferred similarity of those majors may be greater or lower, depending on gender.

For men, ability balance is not a significant predictor of either major diversity or the number of majors, regardless of which sample (all students or survey respondents) is considered. For women, a marginal increase in ability balance increases the likelihood of a double major by

0.06 points at the mean, but this effect is only marginally significant for the set of all female students and becomes insignificant for survey completers. The effect of ability balance on major diversity is consistently insignificant. Thus, ability balance may influence the likelihood of two majors for some students, but its overall impact is considerably smaller in this regard than that of general ability.

With regard to financial impacts, neither of the survey variables is important in predicting either major similarity or the number of majors. The effect of financial aid is consistently insignificant, and the effects of parental income do not present a clear pattern, though it appears that women in the middle and highest parental income brackets may be slightly less likely to double major.

As demonstrated by the graduation-year coefficients, the likelihood of a double major has increased consistently over time. Moreover, the diversity of majors has increased for men over time and may have decreased slightly for women, at least for the full sample. The effect of graduation year is muted or reversed for survey respondents, possibly because these students all graduated in close succession.

Table 5.3: Employment Index, All Cases, by Gender

Gender	Male			Female	
Model	OLS	Outcome	Selection	OLS	Probit
Result			Marginal Effect		Marginal Effect
N	1,539	1,539	6,375	2,543	10,724
R^2/Pseudo R^2	0.02	.	.	0.01	0.02
Rho	.	0.81 (0.05)**	.	.	.
Constant	0.23 (0.05)**	-0.37 (0.09)**	.	0.46 (0.04)**	.
Ability Balance	0.08 (0.05)	0.04 (0.06)	-0.05 (0.04)	-0.04 (0.04)	-0.06 (0.04)*
General Ability	0.05 (0.05)	0.36 (0.07)**	0.44 (0.04)**	-0.13 (0.04)**	0.43 (0.04)**
Resident	0.00 (0.02)	-0.01 (0.02)	-0.01 (0.01)	-0.00 (0.01)	-0.00 (0.01)
Race					
White	.	.	.	.	.
Asian	0.06 (0.03)**	0.03 (0.03)	-0.03 (0.02)	0.05 (0.02)**	-0.04 (0.02)**
Black	0.04 (0.03)	0.04 (0.03)	0.01 (0.02)	-0.01 (0.02)	0.05 (0.01)**
Other	-0.01 (0.04)	-0.03 (0.04)	0.03 (0.03)	-0.03 (0.03)	0.07 (0.03)**
Year of Graduation					
1999-2000	.	.	.	.	.
2001	0.02 (0.03)	0.05 (0.03)	0.03 (0.02)	0.01 (0.02)	0.02 (0.02)
2002	0.02 (0.03)	0.05 (0.03)*	0.06 (0.02)**	-0.03 (0.02)	0.04 (0.02)**
2003	-0.01 (0.03)	0.04 (0.03)	0.08 (0.02)**	-0.04 (0.02)**	0.04 (0.02)**
2004	0.02 (0.03)	0.06 (0.03)**	0.07 (0.02)**	-0.02 (0.02)	0.04 (0.02)**
2005	0.06 (0.02)**	0.12 (0.03)**	0.09 (0.02)**	-0.03 (0.02)	0.05 (0.02)**
2006	0.04 (0.03)*	0.10 (0.03)**	0.09 (0.02)**	-0.04 (0.02)**	0.07 (0.02)**

***=significant at the 10% level; **=significant at the 5% level or below**

Table 5.4: Employment Index, Survey Sample by Gender, No Survey Variables

Gender	Male		Female	
Model	OLS	Probit	OLS	Probit
Result		Marginal Effect		Marginal Effect
N	463	1,623	810	2,917
R^2/Pseudo R^2	0.02	0.02	0.02	
Constant	0.28 (0.10)**	.	0.34 (0.07)**	.
Ability Balance	0.13 (0.11)	-0.03 (0.09)	-0.11 (0.08)	-0.06 (0.07)
General Ability	-0.03 (0.11)	0.49 (0.10)**	-0.04 (0.08)	0.48 (0.08)**
Resident	0.01 (0.03)	-0.06 (0.03)*	-0.02 (0.02)	-0.01 (0.03)
Race				
White	.	.	.	.
Asian	0.12 (0.06)**	-0.08 (0.04)**	0.03 (0.04)	-0.01 (0.04)
Black	-0.01 (0.06)	0.01 (0.05)	0.07 (0.03)**	0.06 (0.03)**
Other	-0.00 (0.06)	-0.00 (0.05)	-0.02 (0.04)	0.06 (0.04)
Year of Graduation				
1999-2004	.	.	.	.
2005	0.05 (0.03)	-0.01 (0.03)	0.05 (0.02)**	0.04 (0.02)
2006	0.03 (0.03)	-0.00 (0.03)	0.03 (0.02)	0.04 (0.02)*

***=significant at the 10% level; **=significant at the 5% level or below**

Table 5.5: Employment Index, Survey Sample by Gender, with Financial Aid

Gender	Male		Female	
Model	OLS	Probit	OLS	Probit
Result		Marginal Effect		Marginal Effect
N	463	1,623	810	2,917
R^2/Pseudo R^2	0.02	0.02	0.02	0.02
Constant	0.28 (0.10)**	.	0.34 (0.07)**	.
Ability Balance	0.13 (0.11)	-0.03 (0.09)	-0.11 (0.08)	-0.07 (0.07)
General Ability	-0.03 (0.11)	0.49 (0.10)**	-0.04 (0.08)	0.48 (0.08)**
Resident	0.01 (0.03)	-0.06 (0.03)*	-0.02 (0.02)	-0.01 (0.03)
Race				
White	.	.	.	.
Asian	0.12 (0.06)**	-0.08 (0.04)**	0.03 (0.04)	-0.01 (0.04)
Black	-0.01 (0.06)	0.01 (0.05)	0.07 (0.03)**	0.07 (0.03)**
Other	-0.00 (0.06)	-0.00 (0.05)	-0.02 (0.04)	0.06 (0.04)
Received Financial Aid	-0.00 (0.03)	0.00 (0.02)	0.01 (0.02)	-0.02 (0.02)
Year of Graduation				
1999-2004	.	.	.	.
2005	0.05 (0.03)	-0.01 (0.03)	0.05 (0.02)**	0.04 (0.02)
2006	0.03 (0.03)	-0.00 (0.03)	0.03 (0.02)	0.04 (0.02)*

***=significant at the 10% level; **=significant at the 5% level or below**

Table 5.6: Employment Index, Survey Sample by Gender, with Parental Income

Gender	Male		Female	
Model	OLS	Probit	OLS	Probit
Result		Marginal Effect		Marginal Effect
N	463	1,623	810	2,917
R^2/Pseudo R^2	0.02	0.03	0.02	0.02
Constant	0.31 (0.11)**	.	0.38 (0.07)**	.
Ability Balance	0.13 (0.11)	-0.03 (0.10)	-0.12 (0.08)	-0.07 (0.07)
General Ability	-0.04 (0.11)	0.50 (0.10)**	-0.02 (0.08)	0.48 (0.07)**
Resident	0.01 (0.03)	-0.07 (0.03)**	-0.03 (0.03)	-0.01 (0.03)
Race				
White	.	.	.	.
Asian	0.13 (0.06)**	-0.08 (0.04)**	0.03 (0.04)	-0.01 (0.04)
Black	-0.02 (0.06)	-0.01 (0.05)	0.07 (0.03)	0.05 (0.03)*
Other	-0.01 (0.06)	-0.01 (0.05)	-0.02 (0.04)	0.06 (0.04)
Year of Graduation				
1999-2004	.	.	.	.
2005	0.05 (0.03)	-0.01 (0.03)	0.05 (0.02)**	0.04 (0.02)*
2006	0.02 (0.03)	-0.00 (0.03)	0.03 (0.02)	0.04 (0.02)*
Parental Income				
Less than $30K	.	.	.	.
$30K - $75	-0.04 (0.06)	-0.05 (0.05)	-0.05 (0.04)	-0.04 (0.03)
$75K - $100	-0.00 (0.06)	-0.03 (0.05)	-0.05 (0.04)	-0.08 (0.03)**
$100K - $150	-0.02 (0.06)	-0.07 (0.05)	-0.03 (0.04)	-0.03 (0.04)
More than $150K	-0.01 (0.06)	-0.08 (0.05)*	-0.06 (0.04)	-0.04 (0.04)

***=significant at the 10% level; **=significant at the 5% level or below**

Table 5.7: Employment Index, Survey Sample by Gender, with Both Survey Variables

Gender	Male		Female	
Model	OLS	Probit	OLS	Probit
Result		Marginal Effect		Marginal Effect
N	463	1,623	810	2,917
R^2/Pseudo R^2	0.03	0.03	0.03	0.02
Constant	0.31 (0.11)**	.	0.38 (0.08)**	.
Ability Balance	0.13 (0.11)	-0.04 (0.10)	-0.12 (0.08)*	-0.07 (0.07)
General Ability	-0.04 (0.11)	0.51 (0.10)**	-0.02 (0.08)	0.49 (0.08)**
Resident	0.01 (0.03)	-0.07 (0.03)**	-0.03 (0.04)	-0.02 (0.03)
Race				
White	.	.	.	.
Asian	0.13 (0.06)**	-0.08 (0.04)**	0.03 (0.04)	-0.01 (0.04)
Black	-0.02 (0.06)	-0.00 (0.05)	0.07 (0.03)**	0.06 (0.03)*
Other	-0.01 (0.06)	0.01 (0.05)	-0.02 (0.04)	0.03 (0.06)
Received Financial Aid	-0.00 (0.03)	-0.01 (0.02)	0.00 (0.02)	-0.03 (0.02)
Year of Graduation				
1999-2004	.	.	.	.
2005	0.05 (0.03)	-0.01 (0.03)	0.05 (0.02)**	0.04 (0.02)**
2006	0.02 (0.03)	-0.00 (0.03)	0.03 (0.02)	0.04 (0.02)**
Parental Income				
Less than $30K	.	.	.	.
$30K - $75	-0.04 (0.06)	-0.05 (0.05)	-0.05 (0.04)	-0.05 (0.03)
$75K - $100	-0.00 (0.06)	-0.03 (0.05)	-0.05 (0.04)	-0.08 (0.03)**
$100K - $150	-0.02 (0.06)	-0.08 (0.05)	-0.03 (0.04)	-0.04 (0.04)
More than $150K	-0.01 (0.06)	-0.09 (0.05)*	-0.06 (0.04)	-0.06 (0.04)*

***=significant at the 10% level; **=significant at the 5% level or below**

5.3 Synthesis of the Empirical Results

When considered simultaneously, the results for the transcript index and for the number and diversity of majors provide an integrated picture of undergraduate skill investment patterns at UNC. In particular, recalling that the decision to double major is actually a decision to be more, rather than less, specialized in the context of the full undergraduate investment, we arrive at the following overall pattern:

- The least generally able individuals tend to acquire one major, and their overall undergraduate investment tends to be relatively narrow.
- As general ability increases, individuals still tend to choose a single major, but they may branch out and take a more diverse range of electives, so that their overall investment is more broad.
- As general ability increases past the midpoint of the distribution, individuals are likely to consider a double major but their overall skill investment pattern is still relatively broad.
- At the top of the general ability distribution, individuals are most likely to make a highly focused investment in two majors. Thus, these individuals are highly specialized despite the fact that they have more than one major. Men choose majors that are less similar as their general ability increases, while the reverse is true for women.
- For ability balance, the picture is similar, except that it is only relevant for females in this sample. The most and least balanced women will tend to be the most specialized, though more balanced individuals may be slightly more likely to choose two majors.

Overall, it appears that there is a critical mass of general ability and/or ability balance that make a broad skill investment possible. Below the threshold, individuals tend to specialize as much as possible and invest in only one major; this may be the limit of what they can achieve. Once that threshold is exceeded, however, individuals first have an incentive to broaden their skill sets, possibly to compensate to some degree for innate imbalances or a lack of innate general ability. Finally, individuals of greatest ability in each dimension may have a smaller incentive to invest broadly because they have greater ability already, and so these individuals may find it most efficient to make a more focused investment in two specific areas that give them distinct employment options.

In particular, the gender difference in the effect of general ability in major similarity patterns for men and women may be partly explained by the fact that both general ability and ability balance influence the overall breadth of the undergraduate skill investment for women but not for men. In this sample, men have a slightly higher transcript index on average than women, indicating that they are slightly more specialized overall. In addition, the women with the least similar set of two majors achieves a degree of major diversity that exceeds that of the men with the greatest major diversity, and the employment index for women is on average slightly higher than that for men. In other words, women appear to make a slightly less specialized investment in general than do men. Thus, while the effect of general ability on the likelihood of a second major is similar across genders, women end up with a more diverse overall investment by virtue of the effect of ability balance. For this reason, it makes sense that highly generally able men would use any additional general ability to decrease the similarity of their majors, while comparable women would choose to become more focused as a counter to their already greater breadth.

Also with regard to gender differences in skill investment, it is interesting to note that ability balance appears to act as a constraint during the skill investment process for women but not for men, as only for women is ability balance a significant predictor of the transcript index. This difference is consistent with the idea that women may be anticipating different labor market outcomes. Women in many professions continue to earn less than men with the same credentials in the same jobs, and women are also more likely to take time out of the labor force or work part-time for family reasons. Thus, the overall return to any given skill may be lower for women, and they also may expect their skills to depreciate. For this reason, women may have an incentive to invest in greater skill breadth, to acquire a wider variety of skills. In this way, they may guarantee themselves a job if they want one and are not too picky about what type of work they do or how much they get paid. And, alternatively, one may ask whether different investment patterns contribute to lower wages for women relative to men, or whether there is some type of mutually reinforcing feedback at work in this regard.

In addition to these effects of ability on the skill investment patterns, the empirical results also suggest that perceived economic uncertainty, or cohort effects, have played an important role in the evolution of skill investment over time. Although the average transcript index does not appear to have changed a great deal over time for most of the years considered, a substantial (.08-.15 courses) and highly significant decrease in this index occurred for both men and women who graduated in 2006; this effect is present in both samples, though it is largest for the survey respondents. While it is not possible to say with certainty what may have caused this large shift in departmental diversification, it is interesting to note that the students who graduated in 2006 would have mostly matriculated in the fall of 2002, just one year after the September 11th, 2001, terrorist attacks on the World Trade Center in New York. It is reasonable to suppose that the economic conditions that were present at that time should have influenced the skill investment decisions of these entering students, while students who were already enrolled at the time of the attacks might have already committed themselves to certain programs of study, given that many general distribution requirements are completed during the first year of college.

Cohort effects also appear to be important for the likelihood of a double major, as the fraction of students graduating with two majors has increased steadily as a result of the elapse of time, even when increases in general ability across admitted cohorts are taken into consideration. This result is consistent with the idea that increasing perceptions of labor market risk have lead more and more students to diversify their skills in a way that would lead to more employment options.

Finally, we can infer that, at least for the selected group of students who completed the survey, financial measures are not particularly important determinants of skill investment patterns. Abilities play a far larger role in this context.[1]

[1]One can imagine, however, that financial matters might contribute to choice of university. If that is the case, and if the skill investment pattern of a given individual would vary across institutions, then we could imagine a substantial role for financial considerations in general, even if we do not observe such a role within the context of a particular school.

5.4 Reconciling Theory and Practice for UNC

Recall that the theoretical part of this paper suggests that more generally able and more balanced individuals will be more likely to graduate with a larger number of skills in the context of considerable perceived uncertainty only if cost complementarities are present in the skill investment process. In particular, the intuition underlying the choice of two skills over one skill has to do with the fact that more generally able individuals find it optimal to acquire a higher level of skill than those who are less generally able. For this reason, more generally able individuals should be more likely to prefer one skill to none but less likely to prefer two to one, given that these individuals make a greater investment per skill. However, under sufficient perceived uncertainty and a decreasing cost per skill, more generally able individuals acquire more skills because they face the greatest marginal surplus from doing so.

Clearly, UNC students who are more generally able are empirically more likely to double major (i.e., to prefer two skills to one), regardless of gender. Thus, a literal application of the theory to this population implies that significant cost complementarities are present. This conclusion is reasonable, given that previous research has also found evidence of such complementarities. Moreover, it appears that general ability plays an extremely important role in determining whether a student chooses to double major, especially when considered beside the effects of other demographic and environmental factors. This observation by itself suggests that a double major is likely to be not only a means toward employment diversification for some students but also an effective market signal of general ability.

However, in interpreting the UNC results, it is instructive also to recognize that the acquisition of a major in any given department is, in essence, the most focused skill investment that undergraduates can make while in college. Students do invest in other skills while in school, they just do so at varying degrees of focus, as suggested by the transcript index. Thus, we can also consider the relevance of the intuition underlying the theoretical model by extending the concept of skill-as-major to that of skill-as-department.

In particular, the effects of general ability and ability balance on the breadth of skill investment, as measured by the transcript index, indicate that the most generally able of both genders and the most balanced women are likely to be among the most specialized. General ability contributes to investment in more skills, but only for those toward the middle of the general ability distribution. Similarly, more ability balance contributes to investment in more skills for most women but has the reverse effect for those who are already extremely balanced.

Thus, we can draw at least two conclusions from the empirical analysis in conjunction with the theory. First, complementarities are most likely present in the learning process. Second, however, it is also the case that breadth of investment, or the overall number of skills acquired, does not necessarily increase with ability. Ability balance contributes to breadth of investment for women, but the most balanced actually become more specialized as their balance increases. This result is consistent with the idea that these most balanced women either find it more costly or have less incentive to invest in a wider range of skills. In addiiton, the most generally able individuals of both genders face a greater return to any particular skill and therefore make a more focused, or more in-depth, investment. These individuals have less need of skill diversification and therefore invest in fewer skills overall.[2]

[2]One important limitation of this analysis is that we do not observe the ability profile of those individuals who did not acquire at least one skill, or who dropped out of school. Thus the observed ability distributions are not necessarily the full ability spectrum for UNC students, and it may be that the most or least balanced or generally able in this sample actually fall closer to the middle of these distributions.

Chapter 6

Conclusions

I began this investigation with the questions of why the fraction of students with two majors has been increasing over time, and of how the interaction of abilities and perceptions of labor market uncertainty may have contributed to this trend. The results of the theoretical and empirical analyses presented here suggest that this trend can be explained (at least in part) as resulting from both an increase in mean SAT scores over time and the response of the most generally able students to increases in their perceived future wage uncertainty.

From a theoretical perspective, increases in perceived uncertainty increase the set of individuals who prefer two skills to one, and those who are most generally able will find it most cost-effective to acquire two skills (rather than opt out of the skilled labor market) in this context. From an empirical perspective, students of higher general ability of both genders are much more likely to graduate with two majors, and the mean ability of graduates from UNC has been increasing steadily from year to year. Moreover, perceived uncertainty, as measured by cohort effects, appears to have been increasing over time as well. Thus, an increasing fraction of students is of the type that is likely to choose two majors, and this type has been given an increasing incentive to do so.

The policy implications of this research become clear when considered in conjunction with recommendations that other researchers have already made concerning double majors and education policy. In particular, Del Rossi and Hersch (2008) present empirical evidence that students who choose two disparate majors (typically one quantitative and one not) will tend to have higher returns than those students who choose a single major in a non-quantitative field, though these returns are not necessarily any greater than for a single quantitative major. Students who combine a business major with a quantitative major receive additional returns over and above that from a single major in either field, but business majors appear to be special in this regard. The authors also find that those students with double majors tend to be more likely to complete graduate school, more likely to undergo subsequent on-the-job training, and more likely to be in occupations that require a broader range and higher level of specific skills, such as originality, negotiation, and social perceptiveness. Because of the temporal relationships of the variables in their analysis, they are not able to determine whether more able individuals select majors with higher returns or whether certain majors provide both higher returns and greater skills. Regardless, the authors conclude that, because the return to any double-major combination is not identical across double-majors, the observed skills most likely come from the majors themselves rather than from underlying, pre-existing ability. In consequence, these authors recommend that more college students be encouraged to attempt a second major.

However, in considering the results of their paper in conjunction with those of my own, I believe that their conclusions and policy suggestions may be counterproductive. In the first place, there is no reason why a signaling motive for double majoring should result in an equalization of returns across all double-major combinations. Rather, since there are differential returns to single majors, we should expect there to be differential returns to double majors, because students of different abilities prefer and select into different majors[1], and these choices are themselves signals as a result. Second, if the empirical results from my analysis of UNC undergraduates are consistent with the investment patterns of students at other universities, then it is clear that investment patterns depend in large part on initial ability conditions, which is probably in part why test scores continue to play such an important role in college admissions. Students may acquire new skills while in college, and these learned skills may provide some return in the labor market in and of themselves, but the choices of major and the number of majors are strong indicators of underlying ability differences. Thus, initial ability

[1] For example, see Arcidiacono (2004).

differences persist through the college experience and into the labor market and translate into wage differentials.

This conclusion is, in itself, not new. But this conclusion is normally drawn in the context of education level, such as whether or not a student acquires a graduate degree: more able students go farther in school and earn more as a result. However, it is no accident that double majors are more likely to obtain a graduate degree or to engage in on-the-job training after graduation. These choices are all just different ways in which individuals demonstrate their underlying intellectual fitness.

Because the choice of a double major does appear to be such a strong signal of ability at matriculation, recommending that more students double major is likely to have adverse effects on those students who are not already doing so. To the extent that ability is a constraint, more individuals will be able to double major only if (1) the requirements for those majors are reduced or (2) less able students spend more time and money to complete their degrees[2]. Either situation would have the effect of increasing the cost and reducing the value of the double major, much the way that the overall increase in volume and heterogeneity of college graduates over the past several decades has eroded the signaling value of the bachelor's degree and pushed up the cost of college tuition. Moreover, those students who would benefit least from such an investment would be spending the most on it. And those students who are currently double majoring as a way of diversifying their employment options and trying to make themselves stand out in the increasingly competitive labor pool would simply find yet some other way to differentiate themselves and maintain their employment edge. Recommending that less able students double major is essentially encouraging them to chase a moving target that they are unlikely ever to hit.

Therefore, I recommend an alternative policy: we should recognize that students with low or very unbalanced test scores begin their college studies at a disadvantage. To the extent that math and verbal skills can be taught, these skills should be emphasized and mastered before college entry. In some cases, deferral of college entry may be advisable. Students who manage

[2] Currently, double majors do not take longer to complete their educations than single majors. See Del Rossi and Hersch (2008).

to enter a university with poor scores should be encouraged to select one single major that they are comfortable with and to use their electives to focus on improving the fundamental skills that they lack and should have learned in high school. Doing so will put them in a better position to attempt one of the more lucrative majors or to enter a graduate program or to take advantage of on-the-job training opportunities. Improving these fundamentals at an earlier age will give more students the skills that they need to self-insure against employment risk in an increasingly competitive and risky labor market.

Because relatively few researchers have considered the causes and implications of diverse specialization for highly skilled workers, several promising avenues exist for future related research. First, the theoretical part of this paper has made the assumption that workers who acquire more skills at a given level are more likely to be employed, for reasons of labor demand diversification. However, this assumption has not been tested. Abstracting from ability signals, it may be that double majors are perceived by employers as less focused than their peers who select one major. Therefore, it is not clear a priori that choosing two majors necessarily improves employment prospects if all credentials are revealed to potential employers.

Second, the empirical analysis in this paper has considered only bachelor's degree recipients at one university. It would be interesting to investigate whether these same patterns hold at other schools and at other degree levels, such as joint professional degree programs. In particular, considering a broader range of individuals across multiple levels of schooling might provide richer variation in ability data that would allow better observation of the effects of ability balance on skill investment patterns.

Third, the differential impact of ability balance on the transcript index for men and women suggests that women may have systematically different skill investment patterns than men. In particular, because the women in this sample do make a slightly less specialized investment overall than men, it would be interesting to investigate whether these differences could in part explain any remaining wage differentials between genders that cannot be explained by the attributes that hiring managers commonly observe during an interview. For example, Mulligan and Rubinstein (2008) find evidence that recent declines in the gender wage gap are consistent with the idea that more able women in the 1990's selected into the labor force, while those of lower ability selected out of it. This selection rule is a reversal of what was observed in the 1970's, as previously the most able women would not have chosen to work. However, the analysis presented here suggests that there may also be educational investment differences across gender that are not readily observed by the casual researcher. Men and women who supposedly have the same job and the same credentials and the same measured intellectual ability may not really have all the same acquired skills, and it is not clear whether women make different skill investments because they anticipate different outcomes or whether the different outcomes derive in part from this subtle difference in skill investment patterns.

Appendix A

Analytic Results and Proofs

Result 1:

The individual optimally acquiring one skill selects skill

$$\theta_{11}^* = 0$$

at level

$$\lambda_{11}^* = g\left(1 + \frac{1}{2\sigma}\right).$$

In addition,

$$\frac{\partial \lambda_{11}^*}{\partial g} > 0,\ \frac{\partial \lambda_{11}^*}{\partial \sigma} < 0,\ \frac{\partial \lambda_{11}^*}{\partial v} = 0,\ \frac{\partial^2 \lambda_{11}^*}{\partial \sigma \partial g} < 0.$$

Proof of Result 1:

An individual acquiring one skill solves

$$\max_{\lambda_{11},\, \theta_{11}} \{S_1 = B_1 - C_{11}\}.$$

The first-order conditions (FOC) in this case are given by

$$\lambda_{11}:\ \frac{g}{\sigma} - 2(\lambda_{11}^* - g) = 0$$

and

$$\theta_{11}:\ -2v\sin(\theta_{11}^*) = 0.$$

Solving these yields

$$\lambda_{11}^* = g\left(1 + \frac{1}{2\sigma}\right)$$

and

$$\theta_{11}^* = 0, \pi.$$

Similarly, the second-order conditions (SOC) are

$$\lambda_{11}: \ -2 < 0$$

and

$$\theta_{11}: \ -2v\cos(\theta_{11}^*),$$

which is negative in the domain of θ only for $\theta_{11}^* = 0$. Differentiation of λ_{11}^* yields

$$\frac{\partial \lambda_{11}^*}{\partial g} = \left(1 + \frac{1}{2\sigma}\right) > 0,$$

$$\frac{\partial \lambda_{11}^*}{\partial \sigma} = \frac{-g}{2\sigma^2} < 0,$$

and

$$\frac{\partial^2 \lambda_{11}^*}{\partial \sigma \partial g} = \frac{-1}{2\sigma^2} < 0.$$

Result 2:

The individual optimally acquiring two skills selects diversity

$$\delta^* = \begin{cases} \pi, & v \leq \frac{1}{2\pi} \\ 2\arcsin\left(\frac{1}{2\pi v}\right), & v > \frac{1}{2\pi} \end{cases}$$

at skill levels

$$\lambda_{21}^* = \lambda_{22}^* = g\left(1 + \frac{1}{4\sigma}\right).$$

Furthermore, for $j = 1, 2$,

$$\frac{\partial \lambda_{2j}^*}{\partial g} > 0,\ \frac{\partial \lambda_{2j}^*}{\partial \sigma} < 0,\ \frac{\partial \lambda_{2j}^*}{\partial v} = 0,\ \frac{\partial^2 \lambda_{2j}^*}{\partial \sigma \partial g} < 0,\ \frac{\partial \delta^*}{\partial g} = 0,\ \frac{\partial \delta^*}{\partial \sigma} = 0,$$

and, for $v > \frac{1}{2\pi}$,

$$\frac{\partial \delta^*}{\partial v} < 0.$$

Proof of Result 2:

An individual acquiring two skills solves

$$\max_{\lambda_{21},\, \lambda_{22},\, \theta_{21},\, \theta_{22}} \{S_2 = B_2 - C_{21} - C_{22}\}.$$

When the individual acquires two skills, the distance δ between the two skills may be defined as $\delta = \theta_{21} - \theta_{22}$, where $\theta_{21} \in [0, \pi]$ and $\theta_{22} \in [-\pi, 0]$. The two skills optimally chosen are never of the same sign, as otherwise the individual could acquire the skill negatively symmetric to one of his choices and be strictly better off, without incurring any additional cost. The FOC for the case in which two skills are acquired are given by

$$\lambda_{21}:\ \frac{g}{2\sigma} - 2(\lambda_{21}^* - g) = 0,$$

$$\lambda_{22}:\ \frac{g}{2\sigma} - 2(\lambda_{22}^* - g) = 0,$$

$$\theta_{21}:\ \frac{1}{\pi} - 2v\sin(\theta_{21}^*) = 0,$$

and

$$\theta_{22}:\ \frac{-1}{\pi} - 2v\sin(\theta_{22}^*) = 0.$$

A solution of these conditions yields

$$\lambda_{21}^* = \lambda_{22}^* = g\left(1 + \frac{1}{4\sigma}\right),$$

$$\theta_{21}^* = \begin{cases} \frac{\pi}{2}, & v \leq \frac{1}{2\pi} \\ \arcsin\left(\frac{1}{2\pi v}\right), & v > \frac{1}{2\pi} \end{cases},$$

and

$$\theta_{22}^* = \begin{cases} -\frac{\pi}{2}, & v \leq \frac{1}{2\pi} \\ -\arcsin\left(\frac{1}{2\pi v}\right), & v > \frac{1}{2\pi} \end{cases}.$$

These imply that

$$\delta^* = \begin{cases} \pi, & v \leq \frac{1}{2\pi} \\ 2\arcsin\left(\frac{1}{2\pi v}\right), & v > \frac{1}{2\pi} \end{cases}.$$

Similarly, for $j = 1, 2$, the SOC in this case are given by

$$\lambda_{2j}: \; -2 < 0$$

and

$$\theta_{2j}: \; -2v\cos(\theta_{2j}^*) \leq 0 \, \forall \, \theta_{2j}^* \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right].$$

Differentiation of the optimal skill levels and diversity yield

$$\frac{\partial \lambda_{21}^*}{\partial g} = \frac{\partial \lambda_{22}^*}{\partial g} = 1 + \frac{1}{4\sigma} > 0,$$

$$\frac{\partial \lambda_{21}^*}{\partial \sigma} = \frac{\partial \lambda_{22}^*}{\partial \sigma} = \frac{-g}{4\sigma^2} < 0,$$

$$\frac{\partial^2 \lambda_{21}^*}{\partial \sigma \partial g} = \frac{\partial^2 \lambda_{22}^*}{\partial \sigma \partial g} = \frac{-1}{4\sigma^2} < 0,$$

and

$$\frac{\partial \delta^*}{\partial v} = \frac{-1}{\pi v^2 \sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}} < 0, v > \frac{1}{2\pi}.$$

The remaining derivatives in the result involve differentiation of a constant and are thus equal to zero.

Result 3:

Let $\hat{v}_{01}$, where $0 \leq \hat{v}_{01} \leq 1$, denote the unique value of v at which individuals are just indifferent between acquiring one skill and acquiring none. Individuals characterized by $v > \hat{v}_{01}$ prefer one skill to none. Moreover,

$$\frac{\partial \hat{v}_{01}}{\partial g} < 0, \; \frac{\partial \hat{v}_{01}}{\partial \sigma} > 0, \; \frac{\partial^2 \hat{v}_{01}}{\partial \sigma \partial g} > 0.$$

In particular,

$$\begin{aligned} \sigma \to \infty \quad &\Rightarrow \quad \hat{v}_{01} \to 1 \\ 0 \leftarrow \sigma \quad &\Rightarrow \quad 0 \leftarrow \hat{v}_{01}. \end{aligned}$$

Proof of Result 3:

One skill is preferable to none if the surplus from optimally acquiring one skill is positive, or if

$$S_1^* > 0,$$

which reduces to

$$v^2 - 2v + 1 - g^2 \left(\frac{1}{\sigma} + \frac{1}{4\sigma^2} \right) < 0.$$

Solution of this inequality yields $1 \succ 0$ for v such that

$$1 - g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}} < v < 1 + g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}}.$$

The indicated upper bound for v always lies outside of the relevant domain, while the lower bound may or may not, depending on the magnitudes of g and σ. Therefore, $1 \succ 0$ for v such that

$$1 - g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}} < v \leq 1.$$

Define

$$\hat{v}_{01} = \begin{cases} 1 - g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}}, & g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}} < 1 \\ 0, & g\sqrt{\frac{1}{\sigma} + \frac{1}{4\sigma^2}} \geq 1 \end{cases}.$$

For values of g and σ that do not imply a corner solution for $\hat{v}_{01}$, implicit differentiation of $S_1^* = 0$ yields

$$\frac{\partial \hat{v}_{01}}{\partial g} = \frac{g}{v-1}\left(\frac{1}{\sigma} + \frac{1}{4\sigma^2}\right) < 0,$$

$$\frac{\partial \hat{v}_{01}}{\partial \sigma} = \frac{-g^2}{2(v-1)}\left(\frac{1}{\sigma^2} + \frac{1}{2\sigma^3}\right) > 0,$$

and

$$\frac{\partial^2 \hat{v}_{01}}{\partial \sigma \partial g} = \frac{-g}{v-1}\left(\frac{1}{\sigma^2} + \frac{1}{2\sigma^3}\right) > 0.$$

Moreover, the limiting behavior of $\hat{v}_{01}$ may be inferred from its evaluation at various values of σ.

Result 4:

Let $\hat{v}_{12}$ and $\hat{v}_{21}$, where $\frac{1}{2\pi} \leq \hat{v}_{12} \leq \hat{v}_{21} \leq 1$, denote the two values of v at which individuals are just indifferent between acquiring two skills and acquiring one. Individuals characterized by $v \leq \frac{1}{2\pi}$ never prefer two skills to one. However, individuals characterized by $v > \frac{1}{2\pi}$ prefer two skills to one if $\hat{v}_{12} < v < \hat{v}_{21}$. Moreover,

$$\frac{\partial \hat{v}_{12}}{\partial g} > 0, \; \frac{\partial \hat{v}_{12}}{\partial \sigma} < 0, \; \frac{\partial^2 \hat{v}_{12}}{\partial \sigma \partial g} < 0$$

and

$$\frac{\partial \hat{v}_{21}}{\partial g} < 0, \; \frac{\partial \hat{v}_{21}}{\partial \sigma} > 0, \; \frac{\partial^2 \hat{v}_{21}}{\partial \sigma \partial g} > 0.$$

In particular,

$$\sigma \to \infty \quad \Rightarrow \quad 0.7373 \leftarrow \hat{v}_{12}, \hat{v}_{21} \to 1$$

$$0 \leftarrow \sigma \quad \Rightarrow \quad \hat{v}_{12} \to 0.9731 \leftarrow \hat{v}_{21}.$$

Proof of Result 4:

An individual prefers two skills to one if the surplus from optimally acquiring two exceeds that from optimally acquiring one, or if

$$S_2^* > S_1^*.$$

In the case that $v \leq \frac{1}{2\pi}$, which corresponds to the set of individuals who find maximum diversity optimal if they acquire two skills, the above expression simplifies to

$$v^2 + 2v + \frac{g^2}{8\sigma^2} < 0,$$

which is a contradiction. In this case, the marginal surpluses from both level and diversity are strictly negative, indicating that the marginal skill is clearly not desirable. Therefore, one skill is always preferred to two for individuals characterized by $v \leq \frac{1}{2\pi}$. Conversely, if $v > \frac{1}{2\pi}$, which corresponds to the set of individuals who do not find maximum diversity optimal if they acquire two skills, the relation $S_2^* > S_1^*$ simplifies to

$$v^2 + 2v + 1 + \frac{g^2}{8\sigma^2} - \frac{2\arcsin\left(\frac{1}{2\pi v}\right)}{\pi} - 4v\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2} < 0,$$

which is no longer clearly a contradiction. The marginal surplus from level remains negative, but that from diversity is no longer necessarily so. A closed-form solution of this inequality is not readily available. However, the general behavior of candidate cutoffs may nonetheless be determined.

For notational convenience, let F_{12} denote the function on the left side of this inequality. The derivative of F_{12} with respect to v,

$$\frac{\partial F_{12}}{\partial v} = 2v + 2 - 4\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2},$$

indicates that F_{12} is non-monotonic in v. In particular, if v_{12a} and v_{12b} respectively denote the smaller and larger positive roots of $\frac{\partial F_{12}}{\partial v}$ on the relevant interval, then

$$\frac{\partial F_{12}}{\partial v} \begin{cases} > 0, & v \in \left(\frac{1}{2\pi}, v_{12a}\right) \bigcup (v_{12b}, 1) \approx (.1592, .1988) \bigcup (0.9731, 1), \\ < 0, & v \in (v_{12a}, v_{12b}) \approx (0.1988, 0.9731). \end{cases}$$

This behavior may be verified via examination of the second-order derivative of F_{12} with respect to v at each of these two critical points, or from inspection of a graph of the function. Analytic solutions for v_{12a} and v_{12b} are extraordinarily long and messy and unlikely to provide the reader with much insight. However, *Mathematica* yields numerical approximations of

$$(v_{12a}, v_{12b}) \approx (0.198835 + 3.58492 * 10^{-17} i, 0.973067 - 3.58492 * 10^{-17} i).$$

The imaginary component of these numbers is extremely small and likely results from approximation error. Moreover, the real components of these roots correspond to the magnitudes that would be expected from the examination of a graph of this derivative. Thus, the best real approximation of these two roots is

$$(v_{12a}, v_{12b}) \approx (0.1988, 0.9731).$$

The above multiplicity of roots initially implies the existence of three potential preference cutoffs in the domain of $v \in \left(\frac{1}{2\pi}, 1\right)$. However, one of these possibilities, namely a value of $v \in \left(\frac{1}{2\pi}, v_{12a}\right)$ may be eliminated through an examination of the limiting behavior of the function F_{12}. First, note that, as $\sigma \to \infty$, the function F_{12} converges to

$$L_{12} \equiv v^2 + 2v + 1 - \frac{2 \arcsin\left(\frac{1}{2\pi v}\right)}{\pi} - 4v\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}.$$

That is, the term in F_{12} containing g and σ effectively disappears, leaving an expression that depends only on the value of v. Evaluation of this limiting function at the value $v = \frac{1}{2\pi}$ yields $L_{12}\left(\frac{1}{2\pi}\right) \approx 0.3436$, which is positive. Second, recall that F_{12} is increasing for $v \in \left(\frac{1}{2\pi}, v_{12a}\right)$. Finally, note that, for any given value of g, decreases in σ shift the function F_{12} vertically upward relative to L_{12}. These three observations imply that a root of F_{12} falling in the interval $\left(\frac{1}{2\pi}, v_{12a}\right)$ does not exist for any value of σ. In other words, F_{12} is strictly positive for all $v \in \left(\frac{1}{2\pi}, v_{12a}\right)$. In consequence, two candidates remain as roots of the function F_{12} for

$v \in \left(\frac{1}{2\pi}, 1\right)$, one or both of which is of interest, depending on the values of g and σ. Consider again the limiting function L_{12} of F_{12} under extremely high levels of uncertainty. Evaluated at $v = 1$, the value of this function is $L_{12}(1) \approx -0.0508$, which is negative. As F_{12}, and thus L_{12}, are increasing for $v \in (v_{12b}, 1)$, no roots of F_{12} exist in this interval for the highest levels of σ. In this case, only one preference cutoff is relevant, namely that in the interval (v_{12a}, v_{12b}), and for $v < 1$ beyond this cutoff two skills are preferred to one. An estimate of this cutoff is roughly $v \approx 0.7373$. However, as uncertainty decreases from its upper limit, it is clear that a second root becomes relevant, as it has already been noted that decreases in uncertainty shift F_{12} vertically. Therefore, for some range of σ, F_{12} will have two roots in $(v_{12a}, 1)$, which converge to the functional min v_{12b} as F_{12} continues to shift upwards. Eventually, for very low values of σ, the function F_{12} will be strictly positive, and one skill will always be preferred to two.

Keeping this general behavior in mind, define two cutoffs, $\hat{v}_{12}$ and $\hat{v}_{21}$, between which two skills are preferred to one, between which the function F_{12} is negative, and where $v_{12a} < \hat{v}_{12} \leq v_{12b}$ and $v_{12b} \leq \hat{v}_{21} \leq 1$. More specifically, let

$$\hat{v}_{12} \begin{cases} \in (v_{12a}, v_{12b}), & \frac{-g^2}{8\sigma^2} \geq L_{12} \\ = v_{12b}, & \frac{-g^2}{8\sigma^2} < L_{12} \end{cases}$$

and

$$\hat{v}_{21} \begin{cases} \in (v_{12b}, 1], & \frac{-g^2}{8\sigma^2} \geq L_{12} \\ = v_{12b}, & \frac{-g^2}{8\sigma^2} < L_{12} \end{cases}.$$

It follows that $\sigma \to \infty$ implies $\hat{v}_{12} \to 0.7373, \hat{v}_{21} \to 1$, while $\sigma \to 0$ implies $\hat{v}_{12} \to v_{12b}, \hat{v}_{21} \to v_{12b}$. Implicit differentiation of the relation $F_{12} = 0$, which both cutoffs must satisfy, yields

$$\frac{\partial v}{\partial \sigma} = \frac{g^2}{4\sigma^3\left(2v + 2 - 4\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)},$$

$$\frac{\partial v}{\partial g} = \frac{-g}{4\sigma^2\left(2v + 2 - 4\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)},$$

and

$$\frac{\partial^2 v}{\partial \sigma \partial g} = \frac{g}{2\sigma^3\left(2v + 2 - 4\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)}.$$

The signs of these derivatives depend on the parenthetical expression in the denominator of each. However, it is clear that the sign of $\frac{\partial v}{\partial \sigma}$ is the same as the sign of $\frac{\partial^2 v}{\partial \sigma \partial g}$, and that the sign of $\frac{\partial v}{\partial g}$ is the opposite of this. If

$$\frac{\partial F_{12}}{\partial v} = 2v + 2 - 4\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2} < 0,$$

which occurs for $v \in (v_{12a}, v_{12b})$, then

$$\frac{\partial v}{\partial \sigma} < 0, \frac{\partial v}{\partial g} > 0, \frac{\partial^2 v}{\partial \sigma \partial g} < 0.$$

Alternatively, if $v \in (v_{12b}, 1)$, then

$$\frac{\partial v}{\partial \sigma} > 0, \frac{\partial v}{\partial g} < 0, \frac{\partial^2 v}{\partial \sigma \partial g} > 0.$$

Therefore,

$$\frac{\partial \hat{v}_{12}}{\partial \sigma} < 0, \frac{\partial \hat{v}_{12}}{\partial g} > 0, \frac{\partial^2 \hat{v}_{12}}{\partial \sigma \partial g} < 0$$

and

$$\frac{\partial \hat{v}_{21}}{\partial \sigma} > 0, \frac{\partial \hat{v}_{21}}{\partial g} < 0, \frac{\partial^2 \hat{v}_{21}}{\partial \sigma \partial g} > 0.$$

Result 5:

Let $\hat{v}_{20}$ and $\hat{v}_{02}$, where $\frac{1}{2\pi} \leq \hat{v}_{20} \leq \hat{v}_{02} \leq 1$, denote the two values of v at which individuals are just indifferent between acquiring two skills and acquiring none. Individuals characterized by $v > \frac{1}{2\pi}$ prefer two skills to none if either $v < \hat{v}_{20}$ or $v > \hat{v}_{02}$. Moreover,

$$\frac{\partial \hat{v}_{20}}{\partial g} > 0, \ \frac{\partial \hat{v}_{20}}{\partial \sigma} < 0, \ \frac{\partial^2 \hat{v}_{20}}{\partial \sigma \partial g} < 0,$$

and

$$\frac{\partial \hat{v}_{02}}{\partial g} < 0, \ \frac{\partial \hat{v}_{02}}{\partial \sigma} > 0, \ \frac{\partial^2 \hat{v}_{02}}{\partial \sigma \partial g} > 0.$$

In particular,

$$\sigma \to \infty \quad \Rightarrow \quad 0.1592 \leftarrow \hat{v}_{20}, \hat{v}_{02} \to 0.8247$$

$$0 \leftarrow \sigma \quad \Rightarrow \quad \hat{v}_{20} \to 0.1611 \leftarrow \hat{v}_{02}.$$

Proof of Result 5:

The proof of this result is similar to that of Result 4. In addition to the three cutoffs defined in the previous results, the overall skill acquisition pattern depends also on the realization of two other cutoffs determining the regions over which two skills are preferred to none, given that $v > \frac{1}{2\pi}$. In particular, in this case two skills are preferred to none if

$$S_2^* > 0,$$

which simplifies to

$$v^2 + 1 - g^2 \left(\frac{1}{2\sigma} + \frac{1}{16\sigma^2} \right) - \frac{\arcsin \left(\frac{1}{2\pi v} \right)}{\pi} - 2v \sqrt{1 - \left(\frac{1}{2\pi v} \right)^2} < 0.$$

For notational convenience, let F_{02} denote the function on the left side of this inequality, and let L_{02} denote the limiting function of F_{02} as $\sigma \to \infty$. Roots of $\frac{\partial F_{02}}{\partial v} = 0$ for $v \in \left(\frac{1}{2\pi}, 1\right)$ are given by

$$(v_{02a}, v_{02b}) = \left(\sqrt{\frac{1}{2} - \frac{\sqrt{1-\frac{1}{\pi^2}}}{2}}, \sqrt{\frac{1}{2} + \frac{\sqrt{1-\frac{1}{\pi^2}}}{2}} \right) \approx (0.1611, 0.9869).$$

Thus,

$$\frac{\partial F_{02}}{\partial v} = 2v - 2\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2} \begin{cases} > 0, & v \in \left(\frac{1}{2\pi}, v_{02a}\right) \bigcup (v_{02b}, 1) \approx (.1592, .1611) \bigcup (0.9869, 1), \\ < 0, & v \in (v_{02a}, v_{02b}) \approx (0.1611, 0.9869). \end{cases}$$

L_{02} clearly behaves in the same manner, as decreases in σ merely shift F_{02} vertically downward relative to L_{02}. Moreover, $L_{02}(1) \approx -0.0508$, which is negative. Therefore, no root of F_{02} exists for $v \in (v_{02b}, 1)$. Two potential roots of F_{02} remain to be considered, namely one in the interval $\left(\frac{1}{2\pi}, v_{02a}\right)$, denoted by $\hat{v}_{20}$, and one in the interval (v_{02a}, v_{02b}), denoted by $\hat{v}_{02}$. For L_{02}, the value of the cutoff in the interval (v_{02a}, v_{02b}) is approximately 0.8247.

Implicit differentiation of $F_{02} = 0$ yields

$$\frac{\partial v}{\partial \sigma} = \frac{-g^2\left(\frac{1}{2\sigma^2} + \frac{1}{8\sigma^3}\right)}{\left(2v - 2\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)},$$

$$\frac{\partial v}{\partial g} = \frac{g\left(\frac{1}{\sigma} + \frac{1}{8\sigma^2}\right)}{\left(2v - 2\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)},$$

and

$$\frac{\partial^2 v}{\partial \sigma \partial g} = \frac{-g\left(\frac{1}{\sigma^2} + \frac{1}{4\sigma^3}\right)}{\left(2v - 2\sqrt{1 - \left(\frac{1}{2\pi v}\right)^2}\right)}.$$

Therefore,

$$\frac{\partial \hat{v}_{20}}{\partial \sigma} < 0, \frac{\partial \hat{v}_{20}}{\partial g} > 0, \frac{\partial^2 \hat{v}_{20}}{\partial \sigma \partial g} < 0$$

and

$$\frac{\partial \hat{v}_{02}}{\partial \sigma} > 0, \frac{\partial \hat{v}_{02}}{\partial g} < 0, \frac{\partial^2 \hat{v}_{02}}{\partial \sigma \partial g} > 0.$$

It follows that $\sigma \to \infty$ implies $\hat{v}_{20} \to \left(\frac{1}{2\pi}\right)$, $\hat{v}_{02} \to 0.8247$, while $\sigma \to 0$ implies $\hat{v}_{20} \to v_{02a}, \hat{v}_{02} \to v_{02a}$.

Result 6:

The individual acquiring two skills in the context of a simultaneous choice cost complementarity (economy of scope) selects diversity

$$\delta^{A*} = \begin{cases} \pi, & v \leq \frac{1}{2\pi\gamma} \\ 2\arcsin\left(\frac{1}{2\pi\gamma v}\right), & v > \frac{1}{2\pi\gamma} \end{cases}$$

at skill levels

$$\lambda_{21}^{A*} = \lambda_{22}^{A*} = g\left(1 + \frac{1}{4\gamma\sigma}\right).$$

Moreover, for $j = 1, 2$,

$$\frac{\partial \lambda_{2j}^{A*}}{\partial \gamma} < 0, \ \frac{\partial^2 \lambda_{2j}^{A*}}{\partial \gamma \partial g} < 0, \ \frac{\partial^2 \lambda_{2j}^{A*}}{\partial \gamma \partial \sigma} > 0,$$

and, for $v > \frac{1}{2\pi\gamma}$,

$$\frac{\partial \delta^{A*}}{\partial \gamma} < 0, \ \frac{\partial^2 \delta^{A*}}{\partial \gamma \partial v} > 0.$$

Proof of Result 6:

The proof of the first part of this result is virtually identical to that of Result 2 and is thus omitted. The additional comparative statics expressions are as follows:

$$\frac{\partial \lambda_{21}^{A*}}{\partial \gamma} = \frac{\partial \lambda_{22}^{A*}}{\partial \gamma} = \frac{-g}{4\gamma^2 \sigma} < 0,$$

$$\frac{\partial^2 \lambda_{21}^{A*}}{\partial \gamma \partial g} = \frac{\partial^2 \lambda_{22}^{A*}}{\partial \gamma \partial g} = \frac{-1}{4\gamma^2 \sigma} < 0,$$

$$\frac{\partial^2 \lambda_{21}^{A*}}{\partial \gamma \partial \sigma} = \frac{\partial^2 \lambda_{22}^{A*}}{\partial \gamma \partial \sigma} = \frac{g}{4\gamma^2 \sigma^2} > 0.$$

In addition, for $v > \frac{1}{2\pi\gamma}$,

$$\frac{\partial \delta^{A*}}{\partial \gamma} = \frac{-1}{\pi v \gamma^2 \sqrt{1 - \left(\frac{1}{2\pi\gamma v}\right)^2}} < 0,$$

$$\frac{\partial^2 \delta^{A*}}{\partial \gamma \partial v} = \frac{1}{\pi \gamma^2 v^2 \left(1 - \left(\frac{1}{2\pi\gamma v}\right)^2\right)^{\frac{1}{2}}} + \frac{1}{4\pi^3 \gamma^4 v^4 \left(1 - \left(\frac{1}{2\pi\gamma v}\right)^2\right)^{\frac{3}{2}}} > 0.$$

Result 7.1:

Individuals with $v \leq \frac{1}{2\pi\gamma}$ facing a simultaneous choice cost complementarity (economy of scope) prefer two skills to one if either

(i) $\gamma \leq \frac{1}{2}$ or

(ii) $\gamma > \frac{1}{2}$, $v < \hat{v}_{21}^A$, and $\sigma \geq \hat{\sigma}_{21}^A$.

Result 7.2:

For $\gamma > \frac{1}{2}$,

$$\frac{\partial \hat{v}_{21}^A}{\partial g} < 0,\ \frac{\partial \hat{v}_{21}^A}{\partial \sigma} > 0,\ \frac{\partial \hat{v}_{21}^A}{\partial \gamma} < 0,\ \frac{\partial^2 \hat{v}_{21}^A}{\partial \gamma \partial g} < 0,\ \frac{\partial^2 \hat{v}_{21}^A}{\partial \gamma \partial \sigma} > 0.$$

Moreover,

$$\sigma \to \infty \quad \Rightarrow \quad \begin{cases} \gamma \to 1 \quad \Rightarrow \quad 0 \leftarrow \hat{v}_{21}^{A} \\ \\ \frac{1}{2} \leftarrow \gamma \quad \Rightarrow \quad \hat{v}_{21}^{A} \to \frac{1}{\pi} \end{cases}$$

$$0 \leftarrow \sigma \quad \Rightarrow \quad 0 \leftarrow \hat{v}_{21}^{A}.$$

Proof of Result 7:

If any values of v exist such that some individuals with $v \leq \frac{1}{2\pi\gamma}$ prefer two skills to one while others do not, these may be found where

$$S_2^{A*} > S_1^{*},$$

which simplifies to

$$v^2(2\gamma - 1) + 2v + 2\gamma - 2 + \frac{g^2}{8\gamma\sigma^2}(2\gamma - 1) < 0 \tag{A.1}$$

in the case that $v \leq \frac{1}{2\pi\gamma}$. For a given value of v satisfying this relation with equality, the following derivatives are implicit:

$$\frac{\partial v}{\partial g} = \frac{-g(2\gamma - 1)}{8\gamma\sigma^2[v(2\gamma - 1) + 1]} < 0, \ \gamma > \frac{1}{2},$$

$$\frac{\partial v}{\partial \sigma} = \frac{g^2(2\gamma - 1)}{8\gamma\sigma^3[v(2\gamma - 1) + 1]} > 0, \ \gamma > \frac{1}{2},$$

$$\frac{\partial v}{\partial \gamma} = \frac{-v^2 - 1 - \frac{g^2}{8\gamma\sigma^2} + \frac{g^2(2\gamma-1)}{16\gamma^2\sigma^2}}{v(2\gamma - 1) + 1} < 0.$$

Moreover,

$$\frac{\partial^2 v}{\partial\gamma\partial\sigma} = \frac{\frac{g^2}{4\gamma\sigma^3} - \frac{g^2(2\gamma-1)}{8\gamma^2\sigma^3}}{v(2\gamma - 1) + 1} > 0,$$

$$\frac{\partial^2 v}{\partial\gamma\partial g} = \frac{\frac{-g}{4\gamma\sigma^2} + \frac{g(2\gamma-1)}{8\gamma^2\sigma^2}}{v(2\gamma - 1) + 1} < 0.$$

In identifying cutoff candidates, consider three cases for the value of γ, namely $\gamma = \frac{1}{2}, \gamma < \frac{1}{2}$, and $\gamma > \frac{1}{2}$. When $\gamma = \frac{1}{2}$, relation (17) simplifies greatly to

$$v < \frac{1}{2}.$$

Moreover, in this case, $\frac{1}{2\pi\gamma} = \frac{1}{\pi} < \frac{1}{2}$. Therefore, all individuals characterized by $v \leq \frac{1}{2\pi\gamma}$ prefer two skills to one at this value of the cost complementarity.

For the other two ranges of γ considered, the analysis is somewhat less straightforward. More generally, the roots of F_{12}^A are given by

$$v = \frac{-1}{2\gamma - 1}\left[1 \mp \sqrt{1 - (2\gamma - 1)\left(2\gamma - 2 + \frac{g^2(2\gamma - 1)}{8\gamma\sigma^2}\right)}\right].$$

In the case that $\gamma < \frac{1}{2}$, it follows from Descartes' Rule of Signs that F_{12}^A has at most two positive real roots, the expressions for which are given above, and that no negative real roots exist. The corresponding parabola opens downward. Therefore, in the event that no positive real roots exist, two skills are always preferred to one for $v \leq \frac{1}{2\pi\gamma}$. If the roots are, in fact, real, then one of these can be eliminated from consideration because it is greater than one. In consequence, define the only candidate cutoff for this range of γ as

$$v_{21}^A \equiv \frac{-1}{2\gamma - 1}\left[1 - \sqrt{1 - (2\gamma - 1)\left(2\gamma - 2 + \frac{g^2(2\gamma - 1)}{8\gamma\sigma^2}\right)}\right].$$

This root is real when

$$1 \geq (2\gamma - 1)\left(2\gamma - 2 + \frac{g^2(2\gamma - 1)}{8\gamma\sigma^2}\right),$$

or when

$$\sigma^2(4\gamma^2 - 6\gamma + 1) \leq \frac{-g^2(2\gamma - 1)^2}{8\gamma}.$$

If $(4\gamma^2 - 6\gamma + 1) \geq 0$, which occurs when $\gamma \leq \frac{3-\sqrt{5}}{4} < \frac{1}{2}$, then no real value of σ exists that satisfies this relation, so v_{21}^A is not real, and two skills are preferred to one for all $v \leq \frac{1}{2\pi\gamma}$. If, on the other hand, $(4\gamma^2 - 6\gamma + 1) < 0$, which occurs for $\frac{3-\sqrt{5}}{4} < \gamma < \frac{1}{2}$, a real v_{21}^A exists for

$$\sigma \geq \sqrt{\frac{-g^2(2\gamma-1)^2}{8\gamma(4\gamma^2-6\gamma+1)}}.$$

The remaining question for this range of γ and σ is then that of how v_{21}^A compares with $\frac{1}{2\pi\gamma}$. Note first that $\frac{\partial v_{21}^A}{\partial \sigma} < 0$. Therefore, v_{21}^A takes on its lowest value when uncertainty is at its maximum. It is straightforward to show that this minimum value is always greater than $\frac{1}{2\pi\gamma}$ for $\frac{3-\sqrt{5}}{4} < \gamma < \frac{1}{2}$. In particular, note that $\sigma \to \infty$ implies that

$$v_{21}^A \to \frac{-1}{2\gamma-1}[1-\sqrt{1-(2\gamma-1)(2\gamma-2)}] \equiv \bar{v}_{21}^A,$$

which is monotonically decreasing in γ over the relevant range, as is $\frac{1}{2\pi\gamma}$. In addition, the former function of γ falls everywhere above the latter over this domain, as can be inferred by evaluating both at the endpoints of the domain considered. For $\gamma = \frac{3-\sqrt{5}}{4}$, the limiting value of $\bar{v}_{21}^A$ is approximately 1.62, while that of $\frac{1}{2\pi\gamma}$ is 0.83. Similarly, when $\gamma = \frac{1}{2}$, the limiting value of $\bar{v}_{21}^A$ is $\frac{1}{2}$ (apply L'Hopital's Rule), while that of $\frac{1}{2\pi\gamma}$ is $\frac{1}{\pi}$. In other words, even when uncertainty is sufficiently high that a positive real cutoff above which one skill is preferred to two exists, this cutoff falls everywhere above the boundary of the set of individuals under consideration, namely those who find maximum diversity optimal in the event that they acquire two skills. Therefore, all individuals characterized by $v \leq \frac{1}{2\pi\gamma}$ prefer two skills to one when $\gamma < \frac{1}{2}$.

Next, consider the case that $\gamma > \frac{1}{2}$. For notational convenience in this case, define $\hat{\sigma}_{21}^A$ as the value of σ that makes the constant term in F_{12}^A equal to zero, or

$$\hat{\sigma}_{21}^A \equiv \sqrt{\frac{-g^2(2\gamma-1)}{16\gamma(\gamma-1)}}.$$

Then, note that Descartes' Rule of Signs indicates that F_{12}^A has at most one positive real root, which is given by v_{21}^A, as defined above. This root is positive and real if

$$1-\sqrt{1-(2\gamma-1)\left(2\gamma-2+\frac{g^2(2\gamma-1)}{8\gamma\sigma^2}\right)}\leq 0,$$

or

$$(2\gamma-1)\left(2\gamma-2+\frac{g^2(2\gamma-1)}{8\gamma\sigma^2}\right)\leq 0.$$

Since the first term on the left side of this inequality is always positive for $\gamma>\frac{1}{2}$, this will only hold if

$$2\gamma-2+\frac{g^2(2\gamma-1)}{8\gamma\sigma^2}\leq 0,$$

which is true for $\sigma\geq\hat{\sigma}_{21}^A$, which was defined above. For those cases in which v_{21}^A exists, now consider its relationship to the term $\frac{1}{2\pi\gamma}$:

If $\gamma>\frac{1}{2}$ and $\sigma\geq\hat{\sigma}_{21}^A$, then it follows that $\frac{\partial v_{21}^A}{\partial\sigma}>0$ and $\frac{\partial v_{21}^A}{\partial\gamma}<0$. Therefore, v_{21}^A must take on its maximum value as $\sigma\to\infty$, in which case v_{21}^A again converges to $\bar{v}_{21}^A$, which is sometimes but not always greater than $\frac{1}{2\pi\gamma}$ for $\gamma\in\left(\frac{1}{2},1\right)$. Similarly, v_{21}^A takes on its minimum value when $\sigma\to\hat{\sigma}_{21}^A$. In particular, in this case $v_{21}^A\to 0$, which is clearly less than $\frac{1}{2\pi\gamma}$.

Therefore, in general for $\gamma>\frac{1}{2}$, define

$$\hat{v}_{21}^A\equiv\min\left\{v_{21}^A,\frac{1}{2\pi\gamma}\right\},$$

such that $0\leq v_{21}^A\leq 1$, and note that two skills are preferred to one for $v<\hat{v}_{21}^A$.

Result 8:

Individuals with $v\leq\frac{1}{2\pi\gamma}$ facing a simultaneous choice cost complementarity prefer two skills to none (economy of scope) if $v<\hat{v}_{20}^A$ and either

(i) $\gamma \leq \frac{1}{2}$ or

(ii) $\gamma > \frac{1}{2}$ and $\sigma \leq \hat{\sigma}_{20}^{A}$.

Moreover,

$$\frac{\partial \hat{v}_{20}^{A}}{\partial \gamma} < 0,\ \frac{\partial \hat{v}_{20}^{A}}{\partial \sigma} < 0,\ \frac{\partial \hat{v}_{20}^{A}}{\partial g} > 0,\ \frac{\partial^2 \hat{v}_{20}^{A}}{\partial \gamma \partial \sigma} > 0,\ \frac{\partial^2 \hat{v}_{20}^{A}}{\partial \gamma \partial g} < 0.$$

In particular,

$$\sigma \to \infty \quad \Rightarrow \quad \begin{cases} \gamma \to \frac{1}{2} & \Rightarrow \quad 0 \leftarrow \hat{v}_{20}^{A} \\ 0 \leftarrow \gamma & \Rightarrow \quad \hat{v}_{20}^{A} \to 1 \end{cases}$$

$$0 \leftarrow \sigma \quad \Rightarrow \quad \begin{cases} \gamma \to 1 & \Rightarrow \quad \frac{1}{2\pi} \leftarrow \hat{v}_{20}^{A} \\ 0 \leftarrow \gamma & \Rightarrow \quad \hat{v}_{20}^{A} \to 1. \end{cases}$$

Proof of Result 8:

Two skills are preferred to none if

$$S_2^{A*} > 0.$$

For those with $v \leq \frac{1}{2\pi\gamma}$, this becomes

$$v^2 + 1 - \frac{1}{2\gamma} - \frac{g^2}{2\gamma\sigma} - \frac{g^2}{16\gamma^2\sigma^2} < 0,$$

which simplifies to

$$v < v_{20}^{A} \equiv \sqrt{\frac{1}{2\gamma} - 1 + \frac{g^2}{2\gamma\sigma} + \frac{g^2}{16\gamma^2\sigma^2}},$$

provided that γ and σ are such that the value of this cutoff is real. It is always real for $\gamma \leq \frac{1}{2}$, as then $\frac{1}{2\gamma} > 1$. However, if $\gamma > \frac{1}{2}$, then a real value of v_{20}^{A} only exists if

$$\frac{1}{2\gamma} - 1 + \frac{g^2}{2\gamma\sigma} + \frac{g^2}{16\gamma^2\sigma^2} \geq 0,$$

or if

$$\sigma \leq \hat{\sigma}_{20}^{A} \equiv \frac{2\gamma g^2 + g\sqrt{4\gamma^2 + \gamma(4g^2 - 2)}}{8\gamma^2 - 4\gamma},$$

since at most one positive real root for σ exists in this case. In addition, note that implicit differentiation of the expression $F_{02}^{A} = 0$ yields

$$\frac{\partial v_{20}^{A}}{\partial \gamma} = \frac{-2v^2 - 2 - \frac{g^2}{8\gamma^2\sigma^2}}{4\gamma v} < 0,$$

$$\frac{\partial v_{20}^{A}}{\partial \sigma} = \frac{\frac{-g^2}{\sigma^2} - \frac{g^2}{4\gamma\sigma^3}}{4\gamma v} < 0,$$

$$\frac{\partial v_{20}^{A}}{\partial g} = \frac{\frac{2g}{\sigma} + \frac{g}{4\gamma\sigma^2}}{4\gamma v} > 0,$$

Furthermore,

$$\frac{\partial^2 v_{20}^{A}}{\partial \gamma \partial g} = \frac{-g}{16\gamma^3\sigma^2 v} < 0,$$

$$\frac{\partial^2 v_{20}^{A}}{\partial \gamma \partial \sigma} = \frac{g^2}{16\gamma^3\sigma^3 v} > 0.$$

Therefore, in the case that $\gamma \leq \frac{1}{2}$, v_{20}^{A} converges to

$$\bar{v}_{20}^{A} \equiv \sqrt{\frac{1}{2\gamma} - 1}$$

as $\sigma \to \infty$. This expression is monotonically decreasing in γ. Moreover, as $\gamma \to \frac{1}{2}$, this limiting value of v_{20}^{A} converges to zero. Similarly, as $\gamma \to 0$, both $\bar{v}_{20}^{A}$ and the value of $\frac{1}{2\pi\gamma}$ approach infinity, indicating that the cutoff will take on the value of the upper limit of the v-domain considered, namely 1.

When $\gamma > \frac{1}{2}$, $v_{20}^{A} \to 0$ as σ approaches the upper limit of its range for which v_{20}^{A} exists, or $\hat{\sigma}_{20}^{A}$.

In both cases for γ, v_{20}^{A} converges to 1 (infinity) as $\sigma \to 0$. Therefore, define

$$\hat{v}_{20}^{A} \equiv \min\left\{v_{20}^{A}, \frac{1}{2\pi\gamma}\right\},$$

such that $0 \leq v_{20}^{A} \leq 1$. Then, two skills are preferred to none for $v < \hat{v}_{20}^{A}$.

The remaining proofs for the extension follow the same methodology as those above and are therefore omitted.

Appendix B

Simulations

Basic model

This section presents several examples of the way in which the number of skills optimally acquired varies with g, v, and σ. The number of skills chosen is evaluated for nine different values of g and v, for each value of σ considered. The first column in each table provides an example of individuals with $v \leq \frac{1}{2\pi}$, while the remaining columns correspond to individuals characterized by $v > \frac{1}{2\pi}$.

Table 1 below illustrates the skill acquisition pattern for the lowest levels of uncertainty (i.e., $\sigma \leq 0.06$). In particular, Table 1 indicates that all individuals acquire one skill for very low uncertainty. In this case, the level of uncertainty is too low to provide disincentive effects for individuals of high ability balance or incentive effects for those of low balance. The expected skill utilization rate is virtually deterministic, so the marginal level surplus from a second skill is large and negative. As a result, all choose full specialization.

TABLE 1: Skill acquisition pattern for $\sigma \leq 0.06$.

$$\begin{array}{c|ccccccccc}
g\backslash v & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
\begin{array}{c}0.1\\0.2\\0.3\\0.4\\0.5\\0.6\\0.7\\0.8\\0.9\end{array} &
\multicolumn{9}{c}{\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}}
\end{array}$$

The next six tables illustrate the way in which the skill acquisition pattern evolves for further increases in uncertainty. Table 2 below indicates that individuals of lowest general ability and highest ability balance are the first to respond to increases in uncertainty by choosing to acquire no skills. In light of the analytic results above, this stems from the facts that these individuals (1) acquire a low level of skill, which translates into a relatively low return, and (2) find skill acquisition very costly: this combination provides a strong disincentive for skill investment.

TABLE 2: Skill acquisition pattern for $\sigma = 0.1$

$$
\begin{array}{c}
g\backslash v \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9
\end{array}
\begin{array}{c}
\begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\end{array}
$$

Tables 3-7 below demonstrate that individuals with lower levels of v generally face disincentive effects from uncertainty, while those with the highest levels of v, who are the least balanced, generally face incentive effects from the same. In other words, increases in uncertainty cause the first group to be less likely to acquire any skills, with this effect most pronounced for individuals of lower general ability, while such increases cause the latter group to be more likely to acquire multiple skills. Extremely high levels of uncertainty discourage skill acquisition by any but the least balanced. More generally able individuals over-invest in skill level and face a higher cost from that investment; as a result, they are less likely to diversify, given that they choose to acquire one skill.

TABLE 3: Skill acquisition pattern for $\sigma = 0.3$

$$
\begin{array}{c c}
g\backslash v & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\end{array}
$$

Table 4: Skill acquisition pattern for $\sigma = 0.7$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	0	0	0	2	2
0.2	0	0	0	0	0	0	0	2	2
0.3	0	0	0	0	0	1	1	2	2
0.4	0	0	0	0	1	1	1	1	2
0.5	0	0	0	1	1	1	1	1	1
0.6	0	1	1	1	1	1	1	1	1
0.7	1	1	1	1	1	1	1	1	1
0.8	1	1	1	1	1	1	1	1	1
0.9	1	1	1	1	1	1	1	1	1

Table 5: Skill acquisition pattern for $\sigma = 2.0$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	0	0	0	0	2
0.2	0	0	0	0	0	0	0	2	2
0.3	0	0	0	0	0	0	0	2	2
0.4	0	0	0	0	0	0	0	2	2
0.5	0	0	0	0	0	0	1	2	2
0.6	0	0	0	0	0	1	1	2	2
0.7	0	0	0	0	1	1	1	2	2
0.8	0	0	0	1	1	1	1	2	2
0.9	0	0	0	1	1	1	1	1	2

Table 6: Skill acquisition pattern for $\sigma = 5.0$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	0	0	0	0	2
0.2	0	0	0	0	0	0	0	0	2
0.3	0	0	0	0	0	0	0	2	2
0.4	0	0	0	0	0	0	0	2	2
0.5	0	0	0	0	0	0	0	2	2
0.6	0	0	0	0	0	0	0	2	2
0.7	0	0	0	0	0	0	1	2	2
0.8	0	0	0	0	0	0	1	2	2
0.9	0	0	0	0	0	1	1	2	2

Table 7 below illustrates the skill acquisition pattern for the highest levels of uncertainty. In particular, most individuals acquire no skills, while those with the very highest levels of v all acquire two. The surplus from skill level eventually approaches zero as uncertainty approaches infinity, and this leaves individuals with the benefit only from diversification. The magnitude of this benefit is small relative to the cost of skill acquisition for most individuals, so they opt out of the skilled labor market, regardless of their degree of general ability.

Table 7: Skill acquisition pattern for $\sigma \geq 50$

$$
\begin{array}{cc}
g\backslash v & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}
\end{array}
$$

Extended model, economy-of-scope

VERY LOW UNCERTAINTY: $\sigma = 0.005$

TABLE 8: Skill acquisition pattern for $\gamma = 0.9, 0.7$

$$\begin{array}{c|ccccccccc}
g\backslash v & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0.9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}$$

TABLE 9: Skill acquisition pattern for $\gamma = 0.5$

$$\begin{array}{c|ccccccccc}
g\backslash v & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.4 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.5 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.6 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.7 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.8 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{array}$$

LOW UNCERTAINTY: $\sigma = 0.05$

TABLE 10: Skill acquisition pattern for $\gamma = 0.9$

$$\begin{array}{c} g\backslash v \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

TABLE 11: Skill acquisition pattern for $\gamma = 0.7$

$$\begin{array}{c} g\backslash v \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\ \begin{pmatrix} 2 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

TABLE 12: Skill acquisition pattern for $\gamma = 0.5$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	2	2	2	2	2	2	2	2	2
0.2	2	2	2	2	2	2	2	2	2
0.3	2	2	2	2	2	2	2	2	2
0.4	2	2	2	2	2	2	2	2	2
0.5	2	2	2	2	2	2	2	2	2
0.6	2	2	2	2	2	2	2	2	2
0.7	2	2	2	2	2	2	2	2	2
0.8	2	2	2	2	2	2	2	2	2
0.9	2	2	2	2	2	2	2	2	2

MODERATE UNCERTAINTY: $\sigma = 1.0$

TABLE 13: Skill acquisition pattern for $\gamma = 0.9$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	2	2	2	2	2
0.2	0	0	0	0	2	2	2	2	2
0.3	0	0	0	0	2	2	2	2	2
0.4	0	0	0	0	2	2	2	2	2
0.5	0	0	0	0	2	2	2	2	2
0.6	0	0	0	1	2	2	2	2	2
0.7	0	0	1	1	2	2	2	2	2
0.8	0	1	1	1	2	2	2	2	2
0.9	1	1	1	1	2	2	2	2	2

Table 14: Skill acquisition pattern for $\gamma = 0.7$

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
g\backslash v & \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{pmatrix}
\end{array}
$$

Table 15: Skill acquisition pattern for $\gamma = 0.5$

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
g\backslash v & \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
2 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{pmatrix}
\end{array}
$$

HIGH UNCERTAINTY: $\sigma = 2.0$

Table 16: Skill acquisition pattern for $\gamma = 0.9$

$$\begin{array}{c|ccccccccc}
g\backslash v & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
\hline
0.1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\
0.2 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\
0.3 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\
0.4 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\
0.5 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\
0.6 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
0.7 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 \\
0.8 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 2 & 2 \\
0.9 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 2 & 2
\end{array}$$

Table 17: Skill acquisition pattern for $\gamma = 0.7$

$$\begin{array}{c|ccccccccc}
g\backslash v & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
\hline
0.1 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.2 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.3 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.4 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.5 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.6 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.7 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.8 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0.9 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{array}$$

Table 18: Skill acquisition pattern for $\gamma = 0.5$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	2	2	2	2	2	2
0.2	2	0	0	2	2	2	2	2	2
0.3	2	2	0	2	2	2	2	2	2
0.4	2	2	0	2	2	2	2	2	2
0.5	2	2	2	2	2	2	2	2	2
0.6	2	2	2	2	2	2	2	2	2
0.7	2	2	2	2	2	2	2	2	2
0.8	2	2	2	2	2	2	2	2	2
0.9	2	2	2	2	2	2	2	2	2

Table 19: Skill acquisition pattern for $\gamma = 0.4$

$g \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	2	2	2	2	2	2	2	2	2
0.2	2	2	2	2	2	2	2	2	2
0.3	2	2	2	2	2	2	2	2	2
0.4	2	2	2	2	2	2	2	2	2
0.5	2	2	2	2	2	2	2	2	2
0.6	2	2	2	2	2	2	2	2	2
0.7	2	2	2	2	2	2	2	2	2
0.8	2	2	2	2	2	2	2	2	2
0.9	2	2	2	2	2	2	2	2	2

VERY HIGH UNCERTAINTY: $\sigma = 6.0$

TABLE 20: Skill acquisition pattern for $\gamma = 0.9$

$g\backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	0	2	2	2	2
0.2	0	0	0	0	0	2	2	2	2
0.3	0	0	0	0	0	2	2	2	2
0.4	0	0	0	0	0	2	2	2	2
0.5	0	0	0	0	0	2	2	2	2
0.6	0	0	0	0	0	2	2	2	2
0.7	0	0	0	0	0	2	2	2	2
0.8	0	0	0	0	0	2	2	2	2
0.9	0	0	0	0	0	2	2	2	2

TABLE 21: Skill acquisition pattern for $\gamma = 0.8$

$g\backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0	0	0	0	2	2	2	2	2
0.2	0	0	0	0	2	2	2	2	2
0.3	0	0	0	0	2	2	2	2	2
0.4	0	0	0	0	2	2	2	2	2
0.5	0	0	0	0	2	2	2	2	2
0.6	0	0	0	0	2	2	2	2	2
0.7	0	0	0	0	2	2	2	2	2
0.8	0	0	0	0	2	2	2	2	2
0.9	0	0	0	2	2	2	2	2	2

Table 22: Skill acquisition pattern for $\gamma = 0.6$

$$
\begin{array}{c|c}
g\backslash v & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{pmatrix}
\end{array}
$$

Table 23: Skill acquisition pattern for $\gamma = 0.4$

$$
\begin{array}{c|c}
g\backslash v & \begin{array}{ccccccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{array} \\
\begin{array}{c} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{array} &
\begin{pmatrix}
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2
\end{pmatrix}
\end{array}
$$

MATLAB code for simulations:

DS1.m

Computes surplus and the number of skills chosen for nine values each of v and g in matrix form, with g on the vertical axis and v on the horizontal.

The value of sigma (s) may be changed as necessary. Used to conduct simulations for the basic model.

$B = zeros(9, 9, 3);$

$s = .05;$

for $i = 1 : 9$

$g = i/10;$

for $j = 1 : 9$

$v = j/10;$

$B0 = 0;$

$B1 = (g^2)/s + (g^2)/(4 * s^2) - (1 - v)^2;$

if $v <= (1/(2 * pi))$

$B2 = (g^2)/s + (g^2)/(8 * s^2) - 1 - 2 * v^2;$

else

$B2 = (g^2)/s+(g^2)/(8*s^2)+(2*asin(1/(2*pi*v)))/pi+-2-2*v^2+4*v*cos(asin(1/(2*pi*v)));$

end

$B(i, j, 1) = B0;$

$B(i, j, 2) = B1;$

$B(i, j, 3) = B2;$

end

end

$[S, I] = max(B, [], 3);$

$I = I - ones(9, 9);$

surplus $= S$

skills $= I$

DS2.m

Used to conduct simulations for cost complementarity with simultaneous choice (economy-of-scope). The variable j represents γ.

$B = zeros(9, 9, 3);$

$s = 0.5;$

$j = .8;$

for $i = 1 : 9$

$g = i/10;$

for $k = 1 : 9$

$v = k/10;$

$B0 = 0;$

$B1 = (g^2)/s + (g^2)/(4 * s^2) - (1 - v)^2;$

if $v <= (1/(2 * pi * j))$

$B2 = (g^2)/s + (g^2)/(8 * j * s^2) + 1 - 2 * j * v^2 - 2 * j;$

else

$B2 = (g^2)/s + (g^2)/(8 * j * s^2) + (2 * asin(1/(2 * pi * j * v)))/pi - 2 * j - 2 * j * v^2 + 4 * v * cos(asin(1/(2 * pi * j * v)));$

end

$B(i, k, 1) = B0;$

$B(i, k, 2) = B1;$

$B(i, k, 3) = B2;$

end

end

$[S, I] = max(B, [], 3);$

$I = I - ones(9, 9);$

surplus$= S$

skills= I

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